STRESS ANALYSIS OF MULTILAYERED COATING SYSTEMS USING A NEW MESHLESS BOUNDARY COLLOCATION METHOD

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Summary A new meshless boundary collocation method based on elasticity theory is developed for the analysis of multi-layered coating systems. The method introduces a concept of the origin intensity factor to isolate the singularity of the fundamental solutions to overcome singular integration and mesh generation in the boundary element method (BEM). The method is successfully applied to stress analysis of multilayered coating problems.

INTRODUCTION

The traditional numerical schemes such as the finite element method (FEM) encounter great challenges in numerial simulation of thin coating problems, since most coatings may be in micro-scale and aspect ratio can be extremely large. The boundary element method (BEM) has long been considered efficient and accurate for modelling thin structures due to the boundary-only discretizations and its semi-analytical nature. However, the standard BEM formulations require mathematically complex and computationally costly numerical singular integration, and its total computational costs are often as expensive as the FEM.

In this study, a novel meshless boundary collocation method, called singular boundary method (SBM), is developed to analyze thin-coated structures. The key idea is to introduce the concept of the origin intensity factors to isolate the singularities of fundamental solutions. Consequently, the SBM remedies the major bottlenecks of the BEM and is meshless and integration free. We also introduce a non-linear transformation, based on sinh function, for nearly singular functions to further improve the numerical accuracy for 2D thin coatings. Our numerical experiments are very encouraging even when the coating thickness is in the orders of 1E-1 to 1E-6, which is sufficient for modelling most thin coatings in the micro-scales.

SBM FORMULATION FOR ELASTOSTATICS PROBLEM

The SBM interpolation formulations for 2D elasticity problems can be expressed as [1, 2]

$$u_i(\boldsymbol{x}^m) = \sum_{n=1, n\neq m}^{N} \alpha_j(\boldsymbol{s}^n) U_{ij}(\boldsymbol{x}^m, \boldsymbol{s}^n) + \alpha_j(\boldsymbol{s}^m) u_{ij}(\boldsymbol{x}^m), \quad \boldsymbol{x}^m \in \Gamma_u, \tag{1}$$

$$t_i(\boldsymbol{x}^m) = \sum_{n=1, n\neq m}^{N} \alpha_j(\boldsymbol{s}^n) T_{ij}(\boldsymbol{x}^m, \boldsymbol{s}^n) + \alpha_j(\boldsymbol{s}^m) t_{ij}(\boldsymbol{x}^m), \quad \boldsymbol{x}^m \in \Gamma_t, \tag{2}$$

where α_j is the unknown coefficients, u_i and t_i represent the displacement and traction components, U_{ij} and T_{ij} denote the corresponding displacement and traction fundamental solutions, $u_{ij}(\boldsymbol{x}^m)$ and $t_{ij}(\boldsymbol{x}^m)$ are defined as the origin intensity factors when the source $\boldsymbol{s}^n$ and collocation $\boldsymbol{x}^m$ points coincide with each other. The origin intensity factors, which isolate the singularity of the fundamental solutions, can be derived out as:

$$u_{ij}(\boldsymbol{x}^m) = \frac{1}{L_m}\int_{\Gamma_m(\boldsymbol{s})} U_{ij}^{(1)}(\boldsymbol{x}^m, \boldsymbol{s})\, d\Gamma_m(\boldsymbol{s}),\; t_{ij}(\boldsymbol{x}^m) = -\sum_{n=1, m\neq n}^{N} T_{ij}^{(1)}(\boldsymbol{s}^n, \boldsymbol{x}^m). \tag{3}$$

For thin structural problems, the following non-linear transformation, based on sinh function [3], is introduced to remove the near singularity of the fundamental solution:

$$x = \eta + b\sinh(k_1 t + k_2),\; t \in [-1,1], \tag{4}$$

where η, b, k_1, and k_2 are coefficients corresponding to the location of the source and collocation points. The (near-) singularity of fundamental solutions have been removed or filtered out using the procedures described above. It should be noted that, compared with the BEM-based methods, the developed method is easy to implement, requires little data preparation, and also, it avoids potentially troublesome extra quadratures to evaluate solutions in the interior of the domain.

NUMERICAL RESULTS AND DISCUSSIONS

Fig. 1 shows a symmetric two-layer coating system [4] under a uniformly distributed load $p=1$ of half-width $A = 1mm$, where h_{c1} is the thickness of the outside coating, h_{c2} the thickness of the inside coating, h_s the substrate

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height, L the structure length. It is assumed here that the structure length, the contact half width and the Poisson's ratio of coatings and substrate are fixed ($h = h_s + h_{c1} + h_{c2} = 20\ mm$, $L = 20\ mm$, $v = 0.3$). In this example, the outside surfaces and interfaces of the SBM model are discretized with 60 and 100 boundary nodes, respectively.

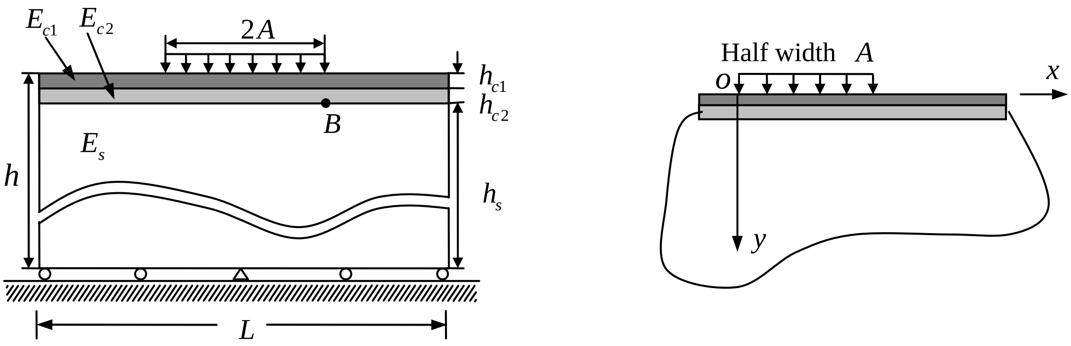

Fig. 1 A multi-coating system under uniform load.

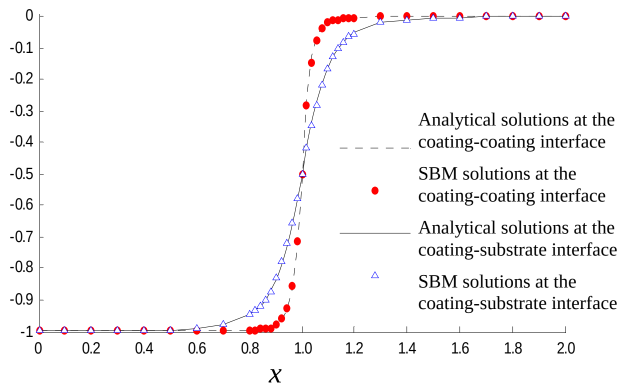

Fig.2 Normal stress (σ_{yy}) prediction.

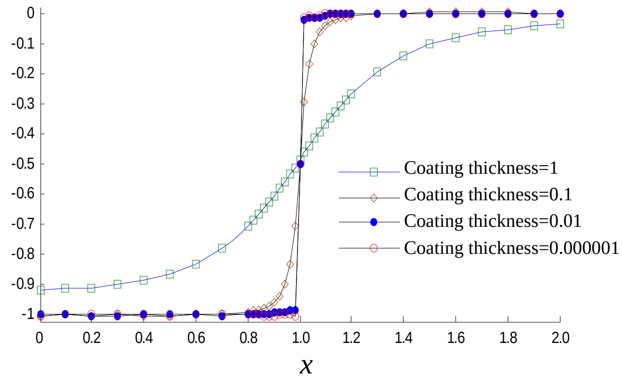

Fig.3 Normal stresses (σ_{yy}) at coating-coating interface.

First, it is assumed that the thickness of both coatings is kept as $0.001\,mm$, and consider the case that both coatings are composed of the same materials as the substrate for which the analytical solution exists. As shown in Fig. 2, the normal (σ_{yy}) stress predictions are in good agreement with the analytical solutions showing high accuracy of the proposed method. Standard BEM formulation, however, as shown in [4], fails to give satisfactory results.

Following, consider the case that both coatings are composed of the different materials as the substrate. The thicknesses of both coatings change from $1\,mm$ to 1E-6 mm. Fig. 3 shows the stress distributions for different coating thickness at the coating-coating interface. While no analytical solution available in this case, we can see that the stress distribution is closer and closer to the loading distribution, which is consistent with the physical interpretation.

Although not presented for the limited space of this paper, it is worth mentioning here that numerous other numerical experiments have been performed and the same observations are made.

CONCLUSIONS

This paper makes the first attempt to extend the meshless SBM to the analysis of 2D multilayered coating systems. Numerical experiments demonstrate that the proposed SBM scheme can well simulate thin coatings even in micro scale thickness by using only a few boundary nodes. And the SBM appears very promising to handle the challenging problems of this type.

References

[1] Chen W., Wang F.Z.: A Method of Fundamental Solutions Without Fictitious Boundary. *Eng Anal Bound Elem* **34** (5):530-532, 2010.

[2] Gu Y., Chen W., Zhang C.Z.: Singular boundary method for solving plane strain elastostatic problems. *Int J Solids Struct* **48** (18):2549-2556, 2011.

[3] Johnston B.M., Johnston P.R., Elliott D.: A sinh transformation for evaluating two-dimensional nearly singular boundary element integrals. *Int J Numer Methods Eng* **69** (7):1460-1479, 2007.

[4] Luo J.F., Liu Y.J., Berger E.J.: Interfacial stress analysis for multi-coating systems using an advanced boundary element method. *Comput Mech* **24** (6):448-455, 2000.