DEVELOPMENT OF CONVOLUTION QUADRATURE BOUNDARY ELEMENT METHOD FOR 2-D POROELASTIC WAVE PROPAGATION

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Summary This paper presents a new time-domain boundary element method in 2-D poroelastic wave propagation. The conventional time-domain BEM approach cannot be used in general, since it is difficult to obtain a closed form of time-domain fundamental solutions in poroelastic wave propagation. To overcome the difficulty, in this research, a new time-domain boundary element method based on the convolution quadrature method is developed for 2-D poroelastic wave propagation. The detail formulation, some numerical and parallelization techniques for fast computation, and numerical results are shown in the presentation.

INTRODUCTION

Since the boundary element method (BEM) is known as a suitable numerical approach for wave analysis, time-domain transient problems have been solved by many researchers using the BEM formulated by Mansur and Brebbia[1], and Hirose[2]. However, it is known that the use of a direct time-domain BEM sometimes causes numerical instability in time-stepping procedures. Recently, the convolution quadrature boundary element method (CQ-BEM) has been researched by several researchers for transient analysis, following the proposal of the convolution quadrature method (CQM) by Lubich[3]. In the formulation of the CQ-BEM, convolution integrals in the time-domain boundary element formulation are numerically approximated by the CQM. Application of the CQM to the time-domain BEM not only improves the numerical stability of the time-domain BEM, but also expands its range of engineering applications, because the CQ-BEM utilizes the Laplace-domain, and not time-domain, fundamental solutions derived from the governing equations of a problem. In fact, the conventional time-domain BEM approach cannot be used for viscoelastic and poroelastic problems because no time-domain fundamental solutions are known. Therefore, using the CQ-BEM is particularly helpful for viscoelastic and poroelastic problems. The CQ-BEM in 2-D viscoelastic wave propagation has worked out so far, then it has accelerated by the fast multipole method (FMM) by Saitoh et al.[4]. In this research, a new boundary element method based on the CQM is formulated for 2-D poroelastic wave propagation.

BEM FORMULATION IN 2-D POROELASTIC WAVE PROPAGATION

The formulation of poroelasticity is based on Biot's theory[5]. Only essential formulas and basic results are shown in 2-D poroelastic wave propagation for plane strain problems. As mentioned in the previous section, the CQ-BEM formulation requires the fundamental solution in Laplace-domain. Laplace-domain fundamental solution $\hat{U}_{ij}(\boldsymbol{x},\boldsymbol{y},s)$ with Laplace parameter s in 2-D poroelastic wave propagation is derived from the following equations:

$$\begin{array}{l} L_{IJ}\hat{U}_{JK}(\boldsymbol{x},\boldsymbol{y},s) \\ = -\delta_{IK}\delta(\boldsymbol{x}-\boldsymbol{y}), \end{array} \quad L_{IJ} = \begin{bmatrix} [L_{ij}] & \{L_{i3}\} \\ \{L_{3j}\}^T & L_{33} \end{bmatrix} = \begin{bmatrix} \left[\dfrac{\partial}{\partial x_k}\dfrac{\partial}{\partial x_l}C_{kilj} - s^2\tilde{\rho}\delta_{ij}\right] & -\left\{\tilde{\alpha}\dfrac{\partial}{\partial x_i}\right\} \\ \left\{\tilde{\alpha}\dfrac{\partial}{\partial x_j}\right\}^T & -\dfrac{1}{s^2\tilde{m}}\dfrac{\partial^2}{\partial x_k^2} + \dfrac{1}{M} \end{bmatrix} \tag{1}$$

where C_{ijkl} is given by $C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta jl + \delta_{il}\delta_{jk})$ with Lamé constants λ and μ. M is Biot modulus, $\tilde{m}$, $\tilde{\alpha}$ and $\tilde{\rho}$ are complex parameters. Small and large indices range from 1 to 2 and 1 to 3, respectively. The integral equation for poroelastic wave propagation in the domain D enclosed by the boundary S can be obtained in time-domain as follows:

$$C_{IJ}(\boldsymbol{x})q_J(\boldsymbol{x},t) = q_I^{\text{in}}(\boldsymbol{x},t) + \int_S U_{IJ}(\boldsymbol{x},\boldsymbol{y},t) * s_J(\boldsymbol{y},t)dS_y - \int_S W_{IJ}(\boldsymbol{x},\boldsymbol{y},t) * q_J(\boldsymbol{y},t)dS_y \tag{2}$$

where t is time, C_{IJ} is the free term, $*$ is the convolution, $q_I^{\text{in}}(\boldsymbol{x},t)$ shows the incident wave. q_I and s_I shows generalized displacement and traction, which are given by $q_I = \{u_1,u_2,p\}^T$ and $s_I = \{t_1,t_2,v\}^T$, respectively, with displacement u_i and traction t_i for solids, and pressure p and flux v for fluids. In addition, $U_{ij}(\boldsymbol{x},\boldsymbol{y},t)$ and $W_{ij}(\boldsymbol{x},\boldsymbol{y},t)$ are time-domain fundamental solution and its double layer kernel, respectively. Normally, the time-domain integral equation (2) is discretized by using the appropriate interpolation functions for the unknown values and solved by a time-stepping algorithm. The boundary integral equation (2), however, cannot be solved using such a scheme because there is no explicit time-domain fundamental solution in 2-D poroelasodynamics. Therefore, the CQM is applied to Eq.(2) to overcome the difficulty.

If we discretize the boundary surface S into M elements by a piecewise constant approximation of the unknown q_I and s_I, taking the limit of $\boldsymbol{x}\in D \to \boldsymbol{x}\in S$, and finally using CQM for the convolutions in Eq.(2), we arrive at the following discretized boundary integral equation for time increment Δt and n steps as follows:

$$\frac{1}{2}q_I(\boldsymbol{x},n\Delta t) = q_I^{\text{in}}(\boldsymbol{x},n\Delta t) + \sum_{\alpha=1}^{M}\sum_{k=1}^{n}\left[A_{IJ}^{n-k}(\boldsymbol{x},\boldsymbol{y}^\alpha)s_J^\alpha(k\Delta t) - B_{IJ}^{n-k}(\boldsymbol{x},\boldsymbol{y}^\alpha)q_J^\alpha(k\Delta t)\right]. \tag{3}$$

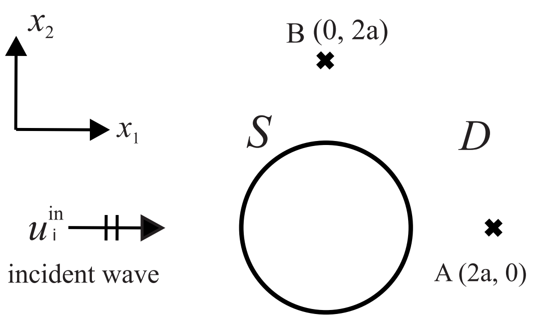

Figure 1. Analysis model.

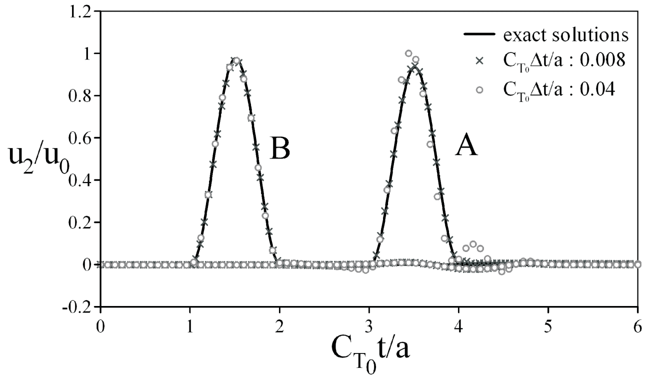

Figure 2. Time variations of u_2/u_0 at A and B in Figure.1.

Here, A^m_{IJ} and B^m_{IJ} are influence functions defined by

$$A^m_{IJ}(\boldsymbol{x},\boldsymbol{y}) = \frac{\mathcal{R}^{-m}}{L}\sum_{l=0}^{L-1}\int_S \hat{U}_{IJ}(\boldsymbol{x},\boldsymbol{y},s^l)e^{-\frac{2\pi iml}{L}}dS_y, \quad B^m_{IJ}(\boldsymbol{x},\boldsymbol{y}) = \frac{\mathcal{R}^{-m}}{L}\sum_{l=0}^{L-1}\int_S \hat{W}_{IJ}(\boldsymbol{x},\boldsymbol{y},s^l)e^{-\frac{2\pi iml}{L}}dS_y \tag{4}$$

where s^l is given by $s^l = \delta(\zeta_l)/\Delta t$ and $\hat{W}_{ij}(\boldsymbol{x},\boldsymbol{y},s)$ is the Laplace-domain traction fundamental solution. In addition, $\mathcal{R}$, L and $\delta(\zeta_l)$ are the CQM parameters[3]. The key of this formulation is that the fundamental solution $\hat{U}_{ij}(\boldsymbol{x},\boldsymbol{y},s)$ in Laplace-domain can be obtained as an explicit form, but not in time-domain, as mentioned before. Therefore, the discretized boundary integral equation (3) can be solved with well known BEM schemes.

PARALLELIZATION FOR LARGE SIZE PROBLEMS

For the practical application, there is a problem to be solved. The computational order of Eq.(4) is $O(M^2L)$. In general, the CQM parameter L is set to be $N = L$ where N shows the total time step. Therefore, it is desirable to decrease the computational costs using some numerical techniques. In this research, distinctive numerical and parallelization techniques using FFT, MPI and GPU for fast computations of Eq.(4) are utilized. For parallelization, the super computer of Tokyo Tech, which is called TSUBAME2.0, is used.

NUMERICAL RESULTS

The scattering problem of an incident plane wave by an object with the radius a and the boundary S in the poroelastic solid D, as shown in Fig.1, is solved by the present method to check the computational accuracy. In this problem, if material parameters of the object are the same as those of the poroelastic solid D, the solutions of the boundary value problems are equivalent to the incident wave $q_I^{\rm in}$. Figure 2 shows time variations of the displacements u_2/u_0 at the points A and B in Fig.1. Solutions using two different time increments, $C_{T0}t/a = 0.04$ and $C_{T0}/a = 0.008$ are plotted and analytical solutions (incident wave $q_I^{\rm in}$) are also shown by the solid line for comparison. Waves in a porous elastic solid saturated by a fluid have frequency dispersion. C_{T0} shows S-wave velocity when no coupling effect between fluid and solid is considered in the problem. We can see that the results obtained by the present method are in good agreement with the analytical solutions. Especially, smaller time step size $C_{T0}/a = 0.008$, which is very smaller than boundary element size, gives more correct results because of the use of the CQM.

CONCLUSIONS

The brief description of the new time-domain BEM in 2-D poroelastic wave propagation was shown in this short paper. The problem of wave scattering by an object in poroelastic media was solved to validate the formulation. In the conference presentation, the detail formulation, some numerical techniques for rapid computation using parallelization techniques, and additional numerical results for a large scale problem will be illustrated.

References

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