A TWO-SCALE APPROACH FOR THE TRANSITION FROM HOMOGENIZATION TO LOCALIZATION IN MULTI-PHASE MATERIALS

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Summary This paper presents a novel method to incorporate the transition from homogenization to localization in a multi-scale framework for heterogeneous materials. The proposed methodology starts from a classical computational homogenization scheme, detects the onset of localization, accommodates the localization band through rotating boundary conditions, enriches the macroscopic problem with a cohesive discontinuity, extracts the constitutive behaviour across the discontinuity from the micro-scale and finally solves both scales in a nested manner up to complete failure.

INTRODUCTION

A vast amount of multi-scale methods for the mechanical behaviour of multi-phase materials have emerged in the past decades. Among these, mixture theories, mean field approaches, asymptotic homogenization schemes, computational homogenization frameworks are a few characteristic categories. They collectively cover the elastic response, visco-elastic, elasto-plastic and generally non-linear material behaviour (in a stable regime). In almost all cases, a scale separation principle has to be adopted. In its simplest form, the principle of separation of scales implies that the spatial variations of the macroscopic fields span a length scale that is much larger than the underlying material volume at the micro scale. This is generally the case if materials are loaded in a stable regime, but it quickly loses its validity at the onset of an instability where significant macroscopic gradients emerge. The most obvious examples thereof is localization of deformation, which is the result of damage and the precursor to fracture. Most of the existing multi-scale approaches cease to be valid and the fundamental question arises if it is possible to adopt a multi-scale strategy beyond the point of loss of stability. This is the main objective of this contribution, whereby use will be made of a computational homogenization scheme prior to localization that gradually transforms in a multi-scale localization-enriched method. From a macroscopic perspective, the results give insight in the constitutive behaviour for damage and fracture, and the continuous-discontinuous transition in particular. The latter is used in many discontinuity-enriched macroscopic solution schemes, whereby the constitutive behaviour has to be assumed up front.

COMPUTATIONAL HOMOGENIZATION

A short review of computational homogenization approaches is first presented [3], highlighting the different schemes and application perspectives: first-order, second-order, beams and shells, thermo-mechanical problems, interfacial problems, multi-physics problems, etc. The intrinsic limitations resulting from the principle of separation of scales are given special attention.

CONTINUOUS-DISCONTINUOUS HOMOGENIZATION-LOCALIZATION

Research questions

The study of the two-scale coupled transition from homogenization to localization is to be performed at different levels [1], whereby different questions arise:

1. How to detect the onset of localization at the level of an initially representative volume element?
2. What is the influence of the boundary conditions used in the stable material regime (e.g. periodic) on the onset of localization and the post-localization regime?
3. How to upscale the mechanical behaviour of the localizing microstructural volume element?
4. What is the macroscopic enrichment needed to incorporate the homogenization-localization enrichment and how is this to be coupled to the microstructural problem?
5. What is the relation with conventional closed-form constitutive models used at the macro-scale for modelling damage and fracture?

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Solution strategy

The answers to the research questions formulated above can be summarized as follows:

- Macroscopic localization is most easily detected through the sign switch of the determinant of the acoustic tensor.
- Conventional boundary conditions have a clear influence on the localization patterns and directions that can develop in a microstructural volume element (MVE). The most frequently used boundary conditions are the periodic ones. The deformation is constrained in a periodic stacking, and hence horizontal and vertical localizations bands are favored. In order to remedy this problem, rotating boundary conditions have been developed, whereby the periodicity concept is applied in a weak sense with respect to directions that are aligned with and orthogonal to the main localization direction [2]. These adaptive boundary conditions are applied from the onset of loading, whereby the orientation of the main deformation band is determined as outlined below for the macroscale problem.
- The scale transition is based on homogenization concepts along the localization band and a discontinuous enrichment perpendicular to the band. The kinematics of the micro-scale problem are extended to weakly impose the overall deformation and the jump as resulting from the macroscale. The fine scale problem is solved, through which the realized jump may differ from the weakly imposed one and whereby the difference between the deformation gradients of the two parts adjacent to the localization band is recovered as well.
- The macroscale problem is recovered through the average stress tensors on each side (incorporating the same traction on the discontinuity), the realized displacement and deformation discontinuity. For the identification of the displacement jump, image analysis algorithms are used. The Hough transform allows to identify a line (band) in the deformation field, giving both its average direction (and width).
- The macroscopic problem is a discontinuity-enriched continuum, for which appropriate solution algorithms can be adopted (e.g. XFEM). The constitutive behaviour emanates from the micro scale solution only.

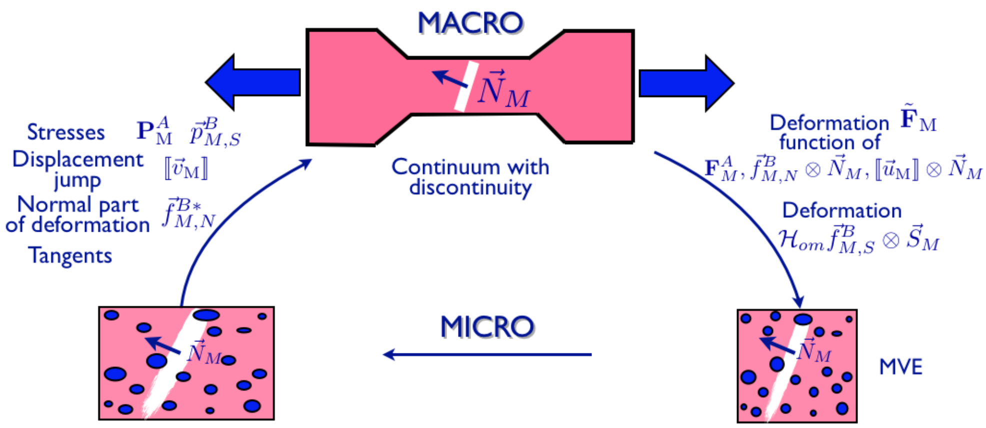

Figure 1. Enriched multi-scale methodology for the homogenization-localization transition in a material volume element

The corresponding enriched multi-scale scheme is depicted in figure 1. An illustrative example of the proposed methodology will be shown.

CONCLUSIONS

This enriched approach incorporates: (1) novel boundary conditions to deal with the onset and progressive localization [2]; (2) a discontinuity-enriched macroscopic description, for which the constitutive behaviour results from the response of the underlying microstructural volume element; (3) a proper algorithm to extract the orientation and magnitude of the discontinuity from the volume element; (4) advanced homogenization concepts along the discontinuity (i.e. parallel to the localization band), whereby a significant computational gain is still realized.

References

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