ATTENUATION OF SOUND WAVE RADIATED BY SPINNING ACOUSTIC DIPOLES IN A CIRCULAR DUCT WITH AN INSERTED IMPEDANCE SURFACE

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Summary This study explores the potential of sound wave scattering in enhancing the side-wall absorption in a circular duct with high frequency sound and high circumferential spinning mode. The eigen-value problem is solved numerically by Chebyshev collocation. The example shows significant potential of absorption enhancement in the frequency range where the normal design of micro-perforated panel on the outer wall is ineffective due to the presence of multiple radial modes. The enhancement is achieved by placing a separator surface near the outer radius. Almost equal sound absorption occurs across the separator surface and by the original wall lining.

1. INTRODUCTION

Duct noise control finds many applications in engineering, ranging from central ventilation noise in buildings, engine inlet and exhaust noise in turbomachines, motor vehicles, as well as in aircraft engines. In an engine, the main aerodynamic noise source is often the interaction between rotor blades and stators, for which a spinning pressure pattern (Tyler & Sofrin 1962) causes dipole noise to radiate to the two sides of the blades. With a special interest in aircraft noise, we focus on circular ducts with a high order of circumferential mode, *m*, while the potentially strong effect of high-speed flow is temporarily set aside. For a given circumferential mode, there are many radial modes that are cut on in the high frequency regime. It is known that the low radial order mode, plane wave being mode 0, is particularly difficult to attenuate and the overall sound absorption rate is determined by the least attenuating mode (Rienstra & Eversman 2001, Sun et al 2008). This study explores the effectiveness of modal scattering for improving sound absorption. As a first step, we consider the eigen-value problem in an infinite duct with hub radius r_{hub} and outer radius r_{tip}. The hard-wall boundary condition at r_{hub} together with the impedance condition at r_{tip} determines the exponential decay rates for all radial modes. We then insert a separator surface $r = r_s$, with $r_{hub} < r_s < r_{tip}$, where the acoustic impedance is defined as

$$Z_s = \frac{p_{r_s-} - p_{r_s+}}{v_{r_s}} = m_s i\omega + R_s \tag{1}$$

Here, *p* is the pressure, *v* is the radial velocity, ω is the angular frequency of sound, m_s, R_s are the mass (per unit surface area) and resistance of the porous separator surface. The question is whether a constant m_s and R_s may be identified such that the decay rate of the least decaying radial mode may be increased for a broad frequency band.

The eigen-value problem is solved numerically using Chebyshev collocation, and the method is summarized in Sec. 2. Modal analysis and other results are given in Sec. 3, while conclusions are drawn in Sec. 4. In the example given, the attenuation rate (per unit distance equal to the outer radius) over an octave band increases from about 8.4 dB to 11 dB.

2. NUMERICAL METHOD

We consider sound propagation along a circular duct between radii $r \in \left[r_{hub}, r_{tip} \right]$. The effect of flow is set aside in this study. Sound propagation follows the classical Helmholtz equation of

$$\left[\partial_{xx} + \partial_{rr} + r^{-1}\partial_r + \left(k^2 - m^2 r^{-2} \right) \right] \phi = 0 \tag{2}$$

Where ϕ is velocity potential, *m* is the circumferential modal index as in $e^{im\theta}$ and $k = \omega / c_0$ is the wavenumber. The radial computational domain is discretized by the Gauss Lobatto grid and the radial derivative ∂_r is replaced by a derivative matrix $[d_r]$. The axial derivative ∂_x is represented by a matrix $[d_x]$ which derives from the discretized Helmholtz equation

$$\begin{aligned} &[d_x]^2 + [d_r]^2 + \left[r^{-1} \right][d_r] + k^2 [I] - m^2 \left[r^{-2} \right] = 0 \\ &[d_x] = i\sqrt{[M]}, \quad [M] = [d_r]^2 + \left[r^{-1} \right][d_r] + k^2 [I] - m^2 \left[r^{-2} \right] \end{aligned} \tag{3}$$

where $\left[r^{-1} \right], \left[r^{-2} \right]$ are the diagonal matrices whose entries are given by r_j^{-1} and r_j^{-2}, respectively, [*I*] is the unit matrix. The matrix square-root is found by eigen-value decomposition,

$$[M]=[P][\Lambda][P]^{-1}, \sqrt{M}=[P]\left[\Lambda^{1/2}\right][P]^{-1}, \quad [\Lambda]=diag\{\lambda_j\}, \tag{4}$$

in which column entries for $[P]$ are the eigen functions corresponding to eigen-value λ_j. For the problem to be well-posed, the dependent values on the two boundaries, ϕ_0, ϕ_N are expressed in terms of the values of inner points $\phi_{j=1\to N-1}$ by solving the two boundary conditions of

$$\partial_r \phi_{hub}=0, \quad \partial_r \phi_{tip}+ikZ_w^{-1}\phi_{tip}=0, \tag{5}$$

where Z_w is the normalized acoustic impedance on the outer wall. The matrix to be diagonalized, $[M]$, has the order of $N-1$. More details may be found in (Huang 2008) for an in-depth analysis of the accuracy of this process used in the construction of out-going wave condition.

When a separator is inserted at radius r_s, the radial domain is divided into two, and they are matched at the separator (of infinitesimal thickness) by the impedance boundary condition, Eq. (1), and the continuity of vertical velocity across the porous separator. Dependent values at all four boundary points are substituted by those at the inner points.

3. RESULTS

Figure 1(a) shows the attenuation rate L_r (dB reduction per unit distance equal to r_{tip}) for two designs: a basic design (dashed line) with the outer wall lined with micro-perforated panel optimized to achieve the highest value of L_r over the frequency band of $f\in[3,8]$, where the dimensionless frequency is defined as $f=\omega r_{tip}/(2\pi c_0)$, and a modified design (solid line) in which a separator has been placed at radius $r_s=0.96r_{tip}$. There is a sharp drop of L_r for the basic design (dashed) between frequency 3.5 and 4.0, signifying the cut-on behaviour of certain radial modes. The circumferential modal index is fixed as m=16. Figure 1(b) shows, for the medium frequency of f=6, the least decaying mode structure with real (solid line) and imaginary (dashed line) parts. Figure 1(c) shows the radial sound intensity distribution. A steep jump at the separator position r_s signifies sound absorption by the separator, which is nearly equal to the absorption by the outer wall. The overall attenuation rates for the two designs are 8.4 and 11.0 dB/r, respectively. The improvements over the the 'cut-on' frequency region of the basic design is between 3 to 5 dB, which is significant.

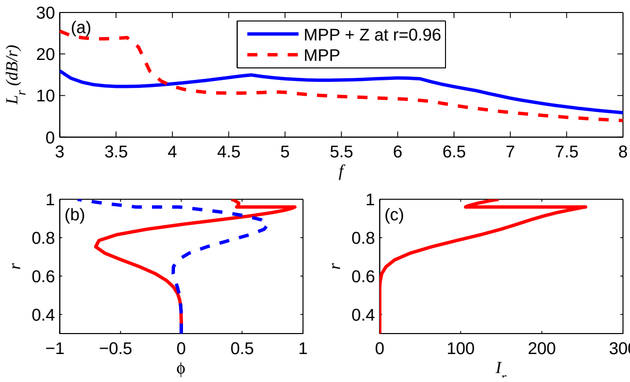

Figure 1. Sound absorption by a micro-perforated panel (MPP) at the outer radius and its enhancement by a separator with absorptive impedance. The MPP is backed by a cavity of depth $0.016r_{tip}$ while the modified has $0.014\,r_{tip}$.

4. CONCLUDING REMARKS

The preliminary example clearly shows significant potential for high-frequency sound absorption by an absorptive separator near the outer radius. The realization of the separator property is left to future studies.

References

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