BAND STRUCTURES OF PHONONIC CRYSTALS WITH WEAKLY BONDED INTERFACES

Ni Zhen[*] [a)], Yue-Sheng Wang[*] & Ch. Zhang[**]
[*] *Institute of Engineering Mechanic, Beijing Jiaotong University, Beijing, 100044, China*
[**]*Department of Civil Engineering, University of Siegen, Siegen, D-57068, Germany*

Summary Band structures of transverse waves propagating in a 2D phononic crystal with weakly bonded interface are computed by using the method based on the Dirichlet-to-Neumann map. We consider a phononic crystal composed of elastic circular cylinders embedded in an elastic matrix in a square lattice. A linear spring layer is used to model the weakly bonded interface between the cylinders and the matrix. The results show that a band gap appears at a very low frequency due to the local resonance.

INTRODUCTION

Phononic crystals [1] (PNCs) are a kind of artificial periodic elastic structures composed of two or more materials with different mass densities and elastic properties. Bloch waves exist due to periodicity in PNCs, making elastic waves in some frequency ranges (i.e. band gaps) cannot pass through the systems. They have received considerable attention as new acoustic functional materials in the last decades because of their potential applications in phononic filters, acoustic lens, noise reduction, vibration damping, etc. In this paper, we will study the wave propagation behaviors in phononic crystals with weakly bonded interfaces. The developed numerical methods, such as the plane-wave expansion method, the transfer matrix method, the multiple scattering theory method, the finite difference time domain method, the finite element method, and the wavelet method, etc., they have been proved to be efficient to calculate the band gaps for some phononic crystals. However, they are difficult in dealing with the weakly bonded interface condition. Therefore, we develop a new method based on the cylindrical wave expansion and Dirichlet-to-Neumann map [2] (shortly denoted as *DtN* map) to calculate the band structures for system.

PROBLEM FORMULATION

Consider a 2D phononic crystal which is composed of circular cylinders embedded in an elastic matrix in a square lattice. The lattice constant is a ; and the holes' radius is r_0 . The Cartesian coordinates and the polar coordinates are related by $\mathbf{x}=(x,y)=(r\cos\theta, r\sin\theta)$. $\Gamma_1 \sim \Gamma_4$ represent the four edges of the square unit cell. A linear spring layer is used to model the weak interface between the cylinders and the host. A harmonic transverse wave polarized in the direction parallel to the cylinders (denoted as *z*-direction) propagates in the *xy*-plane. Then the governing equation of this purely transverse wave can be expressed as $\nabla^2 w_j + k_j^{\,2} w_j = 0$, where the subscript *j*=0 refers to the matrix and *j*=1 to the inclusions, w is displacement component in *z*-direction, $k=\omega/c_t$ is wave number with c_t being the transverse wave velocity and ω the angular frequency. The displacements in the unit cell can be written as a cylindrical wave expansion [3]

$$w_j\left(x,y\right)=\sum_{m=-\infty}^{+\infty} C_m R_m\left(r\right)e^{im\theta},\ j=0,1 \tag{1}$$

where C_m is the undetermined coefficient; $R_m\left(r\right)$ is related to the Bessel functions of the first and second kinds, J_m and Y_m, as

$$R_m\left(r_j\right)=\begin{cases} A_m J_m\left(k_1 r\right), & r<r_0 \\ B_m J_m\left(k_0 r\right)+Y_m\left(k_0 r\right), & r>r_0 \end{cases} \tag{2}$$

where A_m and B_m can be solved by the following interface conditions

$$\sigma_{rz0}(r_0,\theta)=\sigma_{rz1}(r_0,\theta)\,,\quad \sigma_{rz}(r_0,\theta)=K\left[w_0(r_0,\theta)-w_1(r_0,\theta)\right], \tag{3}$$

a) Corresponding author. Email: 08115249@bjtu.edu.cn.

in which $\sigma_{rzj}\left(r,\theta\right)=\mu_j \partial w_j\left(r,\theta\right)/\partial r$ with μ_j being the shear modulus; and K is the stiffness coefficient of the supposed linear spring layer. Then B_m is given by

$$B_m=\frac{sKJ'_m(k_1r_0)Y_m(k_2r_0)-\mu_1k_1J'_m(k_1r_0)Y'_m(k_2r_0)-KJ_m(k_1r_0)Y'_m(k_2r_0)}{KJ_m(k_1r_0)J'_m(k_2r_0)+\mu_1k_1J'_m(k_1r_0)J'_m(k_2r_0)-sKJ'_m(k_1r_0)J_m(k_2r_0)},\quad s=\frac{\mu_1k_1}{\mu_2k_2}. \tag{4}$$

Coefficient A_m is not needed in the following description.
The *DtN* map is an operator that maps the displacement at the edges of the square unit cell to its normal derivative. It can be represented by a matrix Λ such that $\Lambda w=\partial_{\mathbf{n}}w$, where $\mathbf{n}$ is the unit normal vector for each edge of the unit cell. To obtain the *DtN* map, we select *N* pints on each edge of the square unit cell, and thus Λ can be written as a 4*N* $\times$4*N* matrix. Simultaneously, we truncate the series expansions in Eq. (1) by taking *m* from -2*N* to 2*N*-1. Evaluate w and $\partial_{\mathbf{n}}w$ at the 4*N* collocation points of the square unit cell; and write $[w]=\Lambda_1\left\{C_m\right\}$ and $[\partial_{\mathbf{n}}w]=\Lambda_2\left\{C_m\right\}$, separately. Then we can obtain Λ by $\Lambda=\Lambda_2\Lambda_1^{-1}$. Apply the Bloch theorem $f\left(\mathbf{x}+\mathbf{a}\right)=\mathrm{e}^{i\mathbf{k}\cdot\mathbf{a}}f\left(\mathbf{a}\right)$ to the opposite edges of the unit cell where $\mathbf{k}=(k_x,k_y)$ is the Bloch wave vector. The eigenvalue equation can be obtained, from which we are able to calculate the band structures. Consider a system composed of gold cylinders in an epoxy matrix in a square lattice. The stiffness coefficient of the spring layer is $K=3\times10^6\ dyn/cm^3$, the radius of the cylinders is $0.466a$. The band structures are presented in Fig. 1(a). The numerical results show that there is a resonance band gap in the low frequency range, see its amplification in Fig. 1(b). The band gap will become wider and appear at a higher frequency as the spring stiffness coefficient increases, see Fig. 1(c).

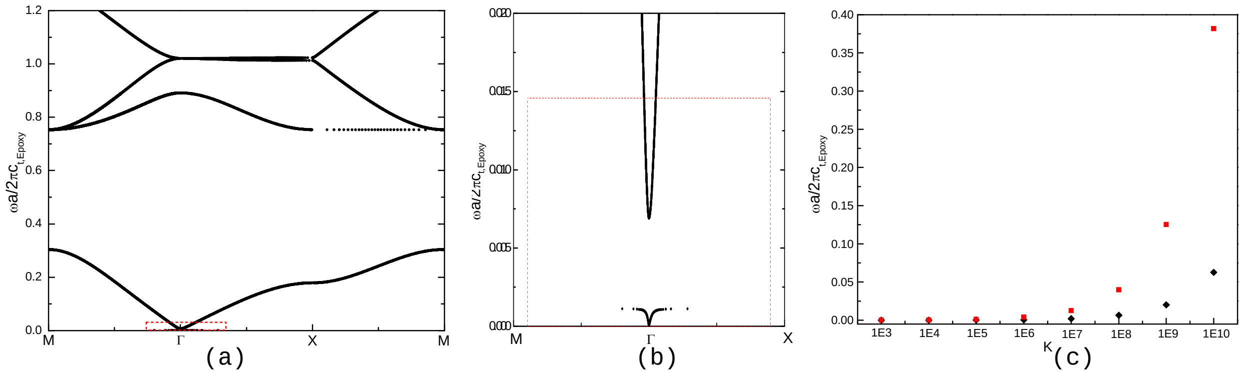

Fig. 1 Band structures of phononic crystal with weakly bonded interface (a); the amplification of the low frequency resonance band gap (b); variation of the resonance band gap with the stiffness coefficient (c).

CONCLUSIONS

The results show that a band gap appears at a very low frequency due to the local resonance. It becomes wider and appears at a higher frequency as the spring stiffness coefficient increases. This provides us a way in tuning the bandgaps of the phononic crystals. The developed method not only can deal with this particular weakly bonded interface condition but also be suitable for various interface conditions between the scatterers and the matrix, with the advantages in accuracy, fast convergence and memory-saving, etc.

References

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