WAVE MOTION AND RESONANCES IN DAMAGED PERIODICALLY LAYERED COMPOSITES

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Summary This paper presents a time-harmonic dynamic analysis of plane elastic wave propagation in periodically layered elastic composites with a single crack or periodically/randomly distributed cracks. The total wave field is written as a sum of the incident wave field and the scattered wave field described by an integral equation approach. The effects of the damage type, location and size, as well as the type and incidence angle of incoming waves are investigated. The focus of the analysis is on the wave transmission and reflection, band-gaps, localization and resonance phenomena due to crack-like damages.

INTRODUCTION

Wave propagation in periodic structures is usually accompanied by band-gaps and localization phenomena, which are observed in photonics, phononics, acoustics and elastodynamics. Band-gaps in phononic crystals have potential applications such as in the design of acoustic filters, vibration control and isolation, noisesuppression and development of new acoustic devices [1]. The phenomena have been discovered theoretically and experimentally verified in periodic elastic structures, and a very good review on the subject including suitable numerical approaches used can be found in [2]. Layered elastic composites are susceptible to damages like cracks and delamination during the manufacturing or in service. In particular, delamination at interfaces between the constituents, and interior cracks in the individual material layers may occur. Such damages could change the dynamic properties of the periodic composites and cause noticeable alterations in band gaps and wave transmission spectra.

Elastic wave propagation in layered elastic composites with a single crack or a distribution of cracks has been analyzed previously by using essentially the dual integral equation method and the hypersingular integral equation method with the crack-opening-displacements (CODs) as unknown quantities. It was shown previously that in most cases the amplitude of the reflected wave field differs not so sufficiently from each other. Thus, in order to simplify the analysis, imperfect interfaces are simulated by spring boundary conditions [3], which might be useful in practical engineering applications.

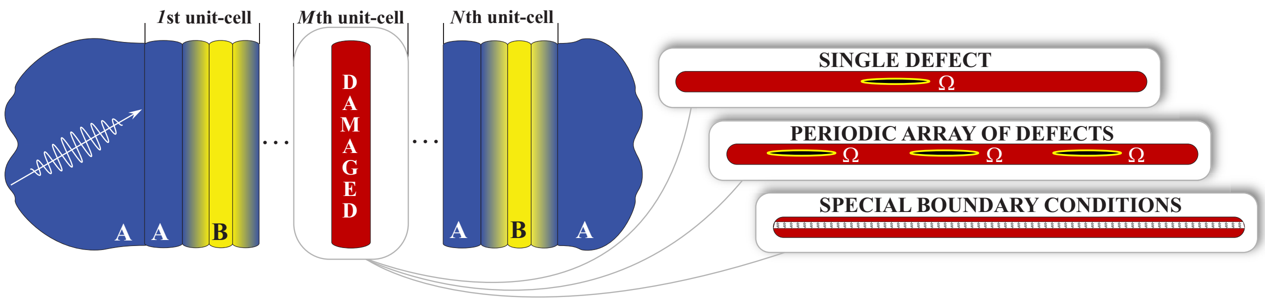

Figure 1. Damaged periodically layered composite.

ANALYSIS AND SIMULATION

Elastic wave diffraction by a single crack or an array of cracks in multilayered composite structures is investigated using boundary integral representations and Green's functions in conjunction with the integral-transform technique [4]. The integral equations derived from the integral representations and the boundary conditions on the crack-faces are solved numerically by a Galerkin method. In the spring model [3], the modified transfer matrix method is used and the spring constant is estimated by the material properties and the crack density. Wave localization in the vicinity of the damages and the structure of the wave field are analyzed by using the stress intensity factors (SIFs) and averaged CODs.

The energy transport of the time-harmonic wave field $\mathbf{u}e^{-i\omega t}$ can be visualized by the energy streamlines, which show the trajectories of the time-averaged energy fluxes [5]. At every point energy streamlines are tangential to the power density vector $\mathbf{e} = (e_x, e_z) = (e_1, e_2)$ introduced by N.A. Umov for elastodynamic fields. The energy transmission coefficient $\kappa^+ = E^+/E^0$ is also introduced as the ratio of the time-averaged energy flow transmitted through the stack of elastic layers E^+ to the energy transferable by the plane wave E^0.

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WAVE SCATTERING, LOCALIZATION AND RESONANCES

In order to investigate the physics of wave scattering, localization and resonances more precisely, the energy streamlines for three typical situations are shown in Figure 2a,b,c. Here the crack is designated by an ellipse. The magnitudes of the resonance motion $|\mathbf{e}|$ are tens times larger in the vicinity of the crack and the energy flow is deviated much more compared to the non-resonant cases (Figure 2a) at the frequency from the first pass-band. An example of wave resonances at the frequency within the band-gaps is given in Figure 2c for the same crack-length $2l$ as shown in Figure 2b. The situation differs from the frequency within the pass-bands: almost no energy flow is observed behind the crack because of the blocking properties of the stack of the periodic elastic layers. For the same reason, the localization of the wave motion appears in the layers nearby the crack and the highest magnitudes of the wave motion are observed near the crack.

For a single crack, the pass-bands of the damaged periodically layered composites are not enlarged and are comparable with that of non-damaged structures. The energy transmission coefficients within the pass-bands have a very similar structure and are independent on the crack distance from the interface d and the crack length $2l$. In the first pass-band, the wave transmission through the stack of layers is almost not influenced by the crack location d in the layer, even though the minima and the maxima of κ^+ are shifted slightly. For a distribution of cracks, two different models are developed to approximate the damaged layer, namely, a periodic array of cracks and continuously distributed springs in the layer. The computed transmission coefficients for the same crack parameters are in a good agreement with each other. The spring model is based on an averaging technique [3] and correspondingly does not predict exactly the same behavior, especially resonance canyons in the pass-bands, but in general it gives very similar patterns.

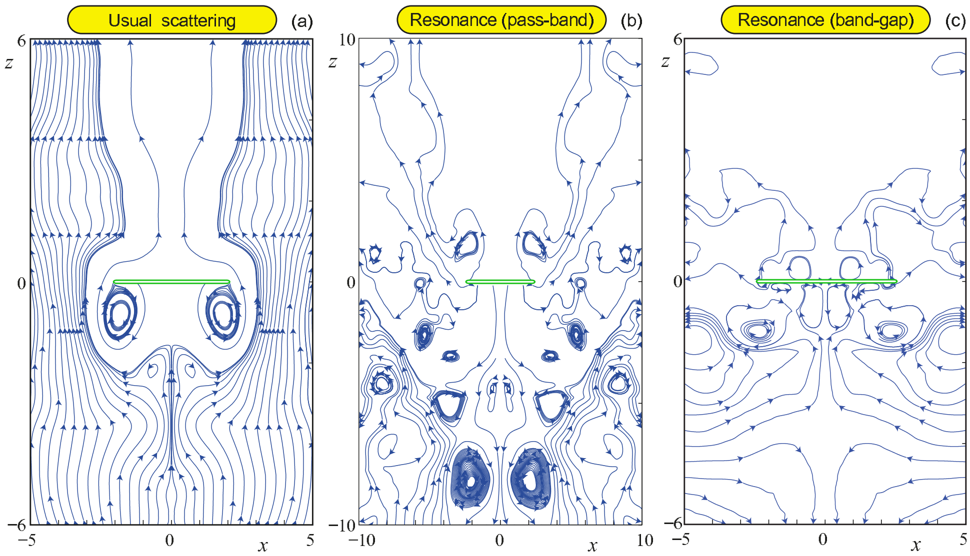

Figure 2. The energy streamlines for plane P-wave propagation at normal incidence.

CONCLUSIONS

The presence of a single crack does not change the band-gaps. The wave propagation characteristics and the crack-characterizing parameters are affected by the crack size, crack location, wave types, wave incidence angle, as well as the material and the geometrical parameters of the periodically layered composites. An interior crack in a periodically layered composite may cause wave localization between the stress-free crack-faces and the interfaces, and it may become stronger when the crack is closer to the interface. An interface crack does not induce wave resonances within the band-gaps. On the other hand, a distribution of cracks may have remarkable influences on the pass-bands and band-gaps.

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