WAVE PROPAGATION IN PIEZOELECTRIC CYLINDERS WITH SURFACE EFFECT

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Summary It has long been recognized that material at or near a boundary usually possesses different properties than the bulk material. The effect of such material heterogeneity can get significant for nano-sized structures. In this work, we investigate the surface effect on wave propagation in a piezoelectric cylinder with radial polarization. To this purpose, we first establish the theory of surface piezoelectricity for a generally curved surface. Different from all available studies, we model the material surface as a thin piezoelectric shell with specified material properties, and derive the governing equations directly from the three-dimensional theory of piezoelectricity. The state-space formulism is employed to give a transfer relation between the state vectors at the top and bottom surfaces of the shell. Using the power series of the transfer matrix, the equations of surface piezoelectricity for different orders are derived. A numerical example is given to demonstrate the surface effect on wave propagation in a piezoelectric cylinder.

INTRODUCTION

With the development of modern technology, the size of structures or components gets smaller and smaller. Many micro- and even nano-sized multifunctional structures and devices have been widely used in science and engineering, and are known as the micro-electro-mechanical systems (MEMS) and nano-electro-mechanical systems (NEMS). In a usual macro-structure, there is no need to consider the material heterogeneity caused by the different environmental constraints imposed on the atoms at different positions (e.g. surface region and bulk region). In MEMS or NEMS, the aspect surface-to-volume ratio is much larger than that of a macro-structure, and the surface may play an important role in determining the mechanical behavior of these tiny systems [1,2].

The surface/interface effects have already been noticed and studied for more than one hundred years since Gibbs' pioneering contribution, with a massive body of literature that can be found on the subject. Of particular relevance to the present work are studies using continuum mechanics methods. Gurtin and Murdoch [3] developed a rigorous mathematical framework for the continuum theory of a deformable material surface. It is shown that the conventional boundary conditions in the classical elasticity are replaced by a set of two-dimensional (2D) differential equations governing the surface deformation [4]. Based on this continuum theory (hereafter referred to as the GM theory), the surface effects on different types of waves have been studied [5-7].

It is interesting to point out that, through a particular procedure following the development of plate theories, Mindlin [8] was able to derive the approximate or effective boundary conditions on the plane boundary of a plate covered with a very thin layer. Tiersten [9] presented a comprehensive study of surface waves guided by thin films and showed that Mindlin's treatment is accurate enough when the wave length is much larger than the thickness of the film. It can be shown that the Mindlin-Tiersten model (hereafter abbreviated as the MT model) is exactly the same as the linearized GM theory [4,7] when the residual surface tension is absent and when the surface parameters are properly defined. The MT model has been subsequently adopted by a few authors in the study of various wave phenomena [10-13].

In this paper, we will present a theory of surface piezoelectricity for a general surface boundary by developing a novel method. The surface is modeled as a thin shell that may be endowed with different material properties than the bulk material. The state-space formalism is employed, which enables an easy derivation of a transfer relation between the state vectors at the top and bottom surfaces of the layer. The transfer matrix, when expressed in terms of power series, naturally provides an asymptotic truncation scheme for the effective boundary conditions, which governs the motion of the piezoelectric material surface. The proposed theory is then used to study the surface effect on wave propagation in piezoelectric cylinders.

BASIC FORMULATIONS

The balance equations governing a three-dimensional piezoelectric body subject to the quasi-static assumption for the electric field are listed as follows:

$$\mathrm{div}\,\mathbf{T} = \rho\ddot{\mathbf{u}}\,,\quad \mathrm{div}\,\mathbf{D} = 0\,, \tag{1}$$

where $\mathbf{T}$, $\mathbf{u}$ and $\mathbf{D}$ are the stress tensor, mechanical displacement vector and electric displacement vector, respectively, ρ is the material density, div means the divergence, and a dot denotes partial differentiation with respect to the time variable *t*. The stress $\mathbf{T}$ and the electric displacement $\mathbf{D}$ can be obtained from the strain $\mathbf{S}$ and electric field $\mathbf{E}$ through the constitutive equations

$$\mathbf{T} = \mathbf{CS} - \mathbf{e}^{\mathrm{T}}\mathbf{E}\,,\quad \mathbf{D} = \mathbf{eS} + \boldsymbol{\varepsilon}\mathbf{E}\,, \tag{2}$$

where T denotes the transpose. The strain tensor $\mathbf{S}$ and electric field $\mathbf{E}$ are computed as

$$\mathbf{S} = \tfrac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)\,,\quad \mathbf{E} = \nabla\phi\,, \tag{3}$$

where ∇ is the gradient. Consider an arbitrary curved shell, of uniform thickness h, with parallel upper and lower surfaces as shown in Fig. 1. We establish a parallel curvilinear system (s_1, s_2, s_3), with s_1 and s_2 being two parametric curves which define the parallel surfaces and coincide with the lines of curvatures of the shell, and s_3 along the common normal to the surfaces. The origin of the system is located on the middle surface of the shell.

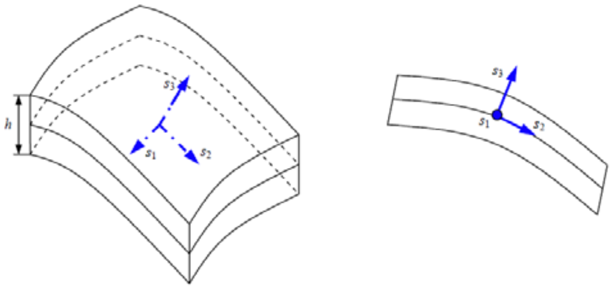

Fig. 1. Curvilinear system for a general curved shell

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The general expressions for various operators in Eqs. (1)-(3) in the established curvilinear system (s_1, s_2, s_3) can be found in Benveniste [14] for example. It is easy to establish the following state equation

$$\partial \mathbf{V} / \partial s_3 = \mathbf{M}(s_1, s_2, \partial_1, \partial_2, \partial_t)\mathbf{V} \,, \tag{4}$$

where $\mathbf{V} = [u_1, u_2, u_3, \phi, T_{13}, T_{23}, T_{33}, D_3]^{\mathrm{T}}$ is the state vector. The formal solution of the above equation can be written as $\mathbf{V} = \exp[\mathbf{M}(s_3 + h/2)]\mathbf{V}(-h/2)$, from which, by using the Taylor's series expression of the matrix exponential, we obtain

$$\mathbf{V}(h/2) = (\mathbf{I} + \mathbf{M}h + \tfrac{1}{2}\mathbf{M}^2 h^2 + \cdots)\mathbf{V}(-h/2) \,, \tag{5}$$

By appropriate truncation and the application of boundary conditions at the upper and lower surfaces of the shell, we can obtain the governing equations for surface piezoelectricity of any order. The derivation is similar to that for a plane boundary as presented in the author's recent paper [15].

When the problem of wave propagation in a piezoelectric cylinder is considered, the state space formalism can also be found in Ref. [16]. Then, we can derive the governing equations for surface piezoelectricity by following Eq. (5). Such governing equations can also be regarded as the effective boundary conditions, which should be used instead of the classical ones at the free surface of the piezoelectric cylinder. The characteristic equation can be derived without difficulty, from which the dispersion relation between the wave number and wave velocity can be determined numerically.

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