SINGULAR BOUNDARY METHOD FOR RADIATION AND WAVE SCATTERING: NUMERICAL ASPECTS AND APPLICATIONS

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Summary The singular boundary method (SBM) in conjunction with Burton and Miller's formulation is applied to some benchmark radiation and wave scattering. Numerical illustrations demonstrate efficiency and accuracy of the present scheme. In particular, this study will also revisit and remedy near trapped mode phenomenon in the multi-body wave propagation.

INTRODUCTION

Despite active research of many decades, numerical solution of the high-frequency and exterior acoustics problems remains a great challenge. The finite element method (FEM) needs to be coupled with some effective special treatments for handling unbounded domain, which are often tricky and largely based on trial-error experiences. The boundary element method (BEM) appears attractive to the exterior acoustics, because the fundamental solutions satisfy the governing equation and the Sommerfeld radiation condition at infinity in advance. However, the BEM [1] encounters computationally expensive singular numerical integration of fundamental solutions. To avoid this troublesome issue, the method of fundamental solution (MFS) distributes the source nodes on a fictitious boundary outside the physical domain and is meshless, integration-free, and highly accurate. However, the placement of fictitious boundary is still an open issue.

Recently, the authors proposed an alternative meshless boundary collocation approach [2,3], singular boundary method (SBM), to solve potential problems. Its key issue is introducing the concept of the source intensity factors to eliminate the singularities of fundamental solutions. This paper presents the SBM coupled with the Burton and Miller's method for radiation and wave scattering to enhance the quality of the solution, particularly in the vicinity of irregular frequencies.

SINGULAR BOUNDARY METHOD WITH BURTON AND MILLER'S FORMULATION

Consider time-harmonic acoustic wave propagation problem in a homogeneous isotropic medium D. Using separation of variables for space and time, it degenerates to the Helmholtz equation $\left(\Delta+k^2\right)u(x)=0,\quad x\in D$. The corresponding SBM approximate solution associated with Burton and Miller's method can be represented as follows

$$u\left(x_m\right)=\sum_{j=1}^{N}\alpha_j\left(G\left(x_m,s_j\right)+\frac{i}{k}\frac{\partial G\left(x_m,s_j\right)}{\partial n_s}\right),\quad q\left(x_m\right)=\frac{\partial u\left(x_m\right)}{\partial n_m}=\sum_{j=1}^{N}\alpha_j\left(\frac{\partial G\left(x_m,s_j\right)}{\partial n_m}+\frac{i}{k}\frac{\partial^2 G\left(x_m,s_j\right)}{\partial n_m\partial n_s}\right),\ x_m\neq s_j \tag{1a}$$

$$u\left(x_m\right)=\sum_{\substack{j=1\\ j\neq m}}^{N}\alpha_j\left(G\left(x_m,s_j\right)+\frac{i}{k}\frac{\partial G\left(x_m,s_j\right)}{\partial n_s}\right)+U^{jj},\ q\left(x_m\right)=\frac{\partial u\left(x_m\right)}{\partial n_m}=\sum_{\substack{j=1\\ j\neq m}}^{N}\alpha_j\left(\frac{\partial G\left(x_m,s_j\right)}{\partial n_m}+\frac{i}{k}\frac{\partial^2 G\left(x_m,s_j\right)}{\partial n_m\partial n_s}\right)+Q^{jj},\ \ x_m=s_j \tag{1b}$$

where N denotes the number of source points s_j, α_j the unknown coefficient, wave number k and $i=\sqrt{-1}$, the fundamental solution $\mathrm{G}\left(x_m,s_j\right)=-\frac{i\pi}{2}H_0^{(1)}\left(k\left\|x_m-s_j\right\|_2\right)$, $H_n^{(1)}$ is the nth order Hankel function of the first kind, the inward normal unit vector n_s and n_m on the source points s_j and collocation points x_m, respectively. If the collocation points and source points coincide, i.e., $x_m=s_j$, the well-known singularities are encountered. The SBM introduces the concept of the source intensity factors U^{jj} and Q^{jj} to avoid these singularities. By utilizing null-field integral equations [4], the source intensity factors in the SBM can be expressed by

$$U^{jj}=U_0^{jj}+\ln\left(\frac{k}{2}\right)+\gamma-\frac{i\pi}{2}+\frac{i}{kL_m}\left(2\pi-\sum_{\substack{j=1\\ j\neq m}}^{N}\frac{\partial G_0\left(x_m,s_j\right)}{\partial n_s}L_j\right),\quad Q^{jj}=-\frac{1}{L_m}\sum_{\substack{j=1\\ j\neq m}}^{N}\left(\frac{\partial G_0\left(x_m,s_j\right)}{\partial n_s}+\frac{i}{k}\frac{\partial^2 G_0\left(x_m,s_j\right)}{\partial n_m\partial n_s}\right)L_j-\frac{k}{4},\ \ x_m=s_j \tag{2}$$

where L_j is the half distance between points s_{j-1} and s_{j+1} defined in Ref. [4], γ Euler constant, $\mathrm{G}_0\left(x_m,s_j\right)=\ln\left(\left\|x_m-s_j\right\|_2\right)$ is the fundamental solution of Laplace equation and the source intensity factors for Laplace equation U_0^{jj} are calculated by the inverse interpolation technique [2,3]. This strategy chooses a sample solution of Laplace equation, e.g., $\bar{u}=x+y+c$, then $2N+1$ linear equations are obtained with $2N+1$ unknowns (U_0^{jj},β_j,c) on N boundary source points and 1 inner point

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$$\bar{u}(x_m)=\sum_{\substack{j=1\\ j\neq m}}^{N}\beta_j G_0(x_m,s_j)+U_0^{jj}+c,\ \frac{\partial \bar{u}(x_m)}{\partial n_m}=\sum_{\substack{j=1\\ j\neq m}}^{N}\beta_j\frac{\partial G_0(x_m,s_j)}{\partial n_m}+\frac{\beta_m}{L_m}\left(2\pi-\sum_{\substack{j=1\\ j\neq m}}^{N}\frac{\partial G_0(x_m,s_j)}{\partial n_s}L_j\right),\ x_m=s_j;\bar{u}(x_m)=\sum_{j=1}^{N}\beta_j G_0(x_m,s_j)+c,\ x_m\neq s_j \tag{3}$$

Therefore, the source intensity factors U_0^{jj} are determined by Eq. (3). Hence the SBM approximation (1) is non-singular.

NUMERICAL ASPECTS AND APPLICATIONS

We first investigate the efficiency and accuracy of the present SBM on some benchmark examples. Herein the figure (top-left) shows the SBM convergence rate for plane wave scattering problem of a rigid infinite circular cylinder [5]. Then the method is applied to wave-structure interactions [6]. Consider the incident plane wave propagating along the horizontal direction with arrays of 10 evenly spaced vertical piles at certain frequencies *ka=4.08482*, near trapped mode phenomenon is revisited in the wave structure, and the maximum amplitude appearing on the inner sides of the cylinders is about 150 times over the incident wave amplitude as shown in the figure (middle). Finally, we are absorbed in two effects of disordered arrays and arrays with porous piles on reducing the peak forces associated to trapped modes and regularizing the transmission coefficient for wave propagating through the arrays. Hereupon the figure (right) presents the effect of arrays with unevenly spaced cylinders by displacing randomly from a regular array via a disorder parameter $\lambda=0.1$. Moreover, the figure (bottom-left) plots the horizontal force ratio versus wave number *ka* by regular and irregular arrays ($\lambda=0.1$). Numerical simulations predict that the disordered treatment is an efficient remedy to reduce the wave run-up and wave force.

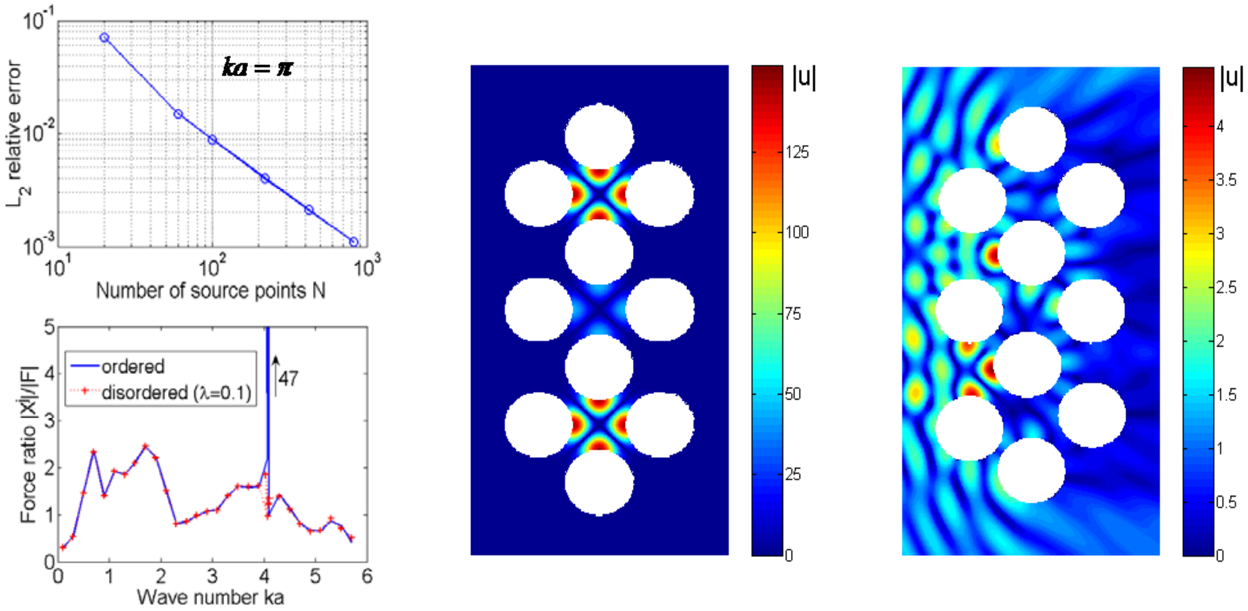

Fig. 1 the SBM convergence rate for plane wave scattering (top-left); the plane wave propagating (*ka=4.08482)* through the arrays of 10 evenly spaced piles (middle); the effect of arrays with unevenly spaced cylinders via a disorder parameter $\lambda=0.1$ (right); the horizontal force ratio on the left-bottom cylinder with regular and irregular arrays ($\lambda=0.1$) (bottom-left).

CONCLUSIONS

Numerical experiments reveal that the SBM is a competitive numerical technique for radiation and wave scattering. The method is mathematically simple, easy-to-program, meshless and integration-free, and has certain advantages on the modelling of high-frequency and exterior wave scattering. On the other hand, the SBM interpolation matrix, however, is fully populated and can be computationally very expensive for large scale problems. We are now implementing the SBM in conjunction with fast algorithms such as the fast multipole and precorrected-FFT methods, to simulate large-scale radiation and wave scattering problems.

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