COMPLETE BANDGAPS IN THREE-DIMENSIONAL HOLEY PHONONIC CRYSTALS WITH RESONATORS

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Summary In this paper we study the bandgap properties of three-dimensional holey phononic crystals with resonators using the finite element method. Complete bandgap can be obtained by the careful design of the geometry of the holes to generate the local resonance of the unit cell. The influence of the geometry parameters on the bandgaps is discussed, and spring-mass models are developed to predict the frequencies of the band edges. The study in this paper is relevant to the optimal design of the bandgaps in light porous materials.

INTRODUCTION

A phononic crystal, a special type of inhomogeneous materials with periodic variations in their elastic properties, has attracted a rapidly growing interest in the past decade. Due to its unique acoustic property of bandgaps within which the propagation of elastic/acoustic waves is fully prohibited, the phononic crystal may have potential applications in acoustic filtering, noise suppression, vibration isolation, and design of new acoustic devices, etc. It has become one of the active fast-developing fields in physics, acoustics, mechanics, material science, etc.

At present, a certain degree of interests is being generated on the phononic crystals with periodic holes (vacuum or filled with air) with their bandgaps mainly determined by the geometric parameters. Most of the reports are focused on the systems with convex holes (circular or regular polygon holes [1]). In a recent paper [2], we proposed a kind of phononic crystals with the cross-like (a kind of non-convex) holes. Resonant unit cells may be formed by the careful design of the geometry of the holes. Therefore wider and lower bandgaps are easily obtained due to the local resonance of the unit cell. In this paper, we extend our work to study three-dimensional holey phononic crystals with resonators in simple cubic lattice. The finite element method (FEM) based on the Comsol Multiphysics software [2] is used to study the effects of the geometric parameters on the bandgap. Spring-mass models are developed by analyzing the eigenmodes at the band edges to examine the origin of the bandgap.

THEORY

The schematic of a proposed phononic crystal is shown in Fig.1. It contains three parts: the external frame (with the inner cubic length *b*), the six connectors with the square cross-section (with the square length *d*) and the large lump (with the cubic length *c*). The lattice constant is *a*. All the other parts inside the frame are vacuum, those are the connected holes. Furthermore, we show the cross-section of the unit cell in Fig. 1(b). The shadowed part is the solid matrix.

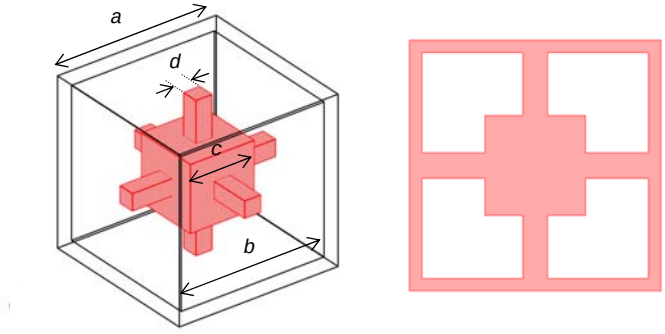

Figure.1 The unit cell of three-dimensional phononic crystals in square cubic lattice (a) and the corresponding cross-section (b).

Due to the periodicity of the phononic crystals, the calculation can be implemented in a representative unit cell. The unit cell is meshed and divided into finite elements connected by nodes. The discrete form of the eigenvalue equations in the unit cell can be written as:

$$(\mathbf{K}-\omega^2\mathbf{M})\mathbf{U}=0\,, \qquad (1)$$

where ω is the circular frequency, $\mathbf{U}$ is the displacement vector; and $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices of the unit cell, respectively. The Bloch theorem should be applied on the boundaries of the unit cell in the

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direction where periodicity applies, yielding for instance the following relationship between the displacements $\mathbf{U}(\mathbf{r})$ for nodes lying on the boundary of the unit cell:

$$\mathbf{U}(\mathbf{r}+\mathbf{a}) = e^{i(\mathbf{k}\cdot\mathbf{a})}\mathbf{U}(\mathbf{r}) . \qquad (2)$$

Then Comsol Multiphysics 3.5a is utilized to directly solve the eigenvalue equation (1) under the complex boundary condition (2). The band structures will be obtained by varying the wave vector $\mathbf{k}$ along the edges of the irreducible Brillouin zone.

RESULTS AND DISCUSSIONS

In this section we present detailed numerical results of the band structures for various systems with different geometry parameters and analyze the mechanism of the appearance of the complete bandgaps. The elastic parameter chosen for the calculations is the Poisson's ratio $\nu = 0.25$, the only material parameter related with the bandgap. In order to show the new features appearing in the band structures of the proposed phononic crystal, we illustrate the dispersion curves for b/a=0.9, c/a=0.8, and d/a=0.1 in Fig. 2(c). For comparison, we also give the band structures of the phononic crystals with cubic hole for b/a=0.9 in Fig. 2(a) and spherical hole for r/a=0.45 in Fig. 2(b). The vertical axis is the normalized frequency $\Omega = \omega a/2\pi c_t$ (with c_t being the transverse wave velocity of the matrix). The horizontal axis is the reduced wave number.

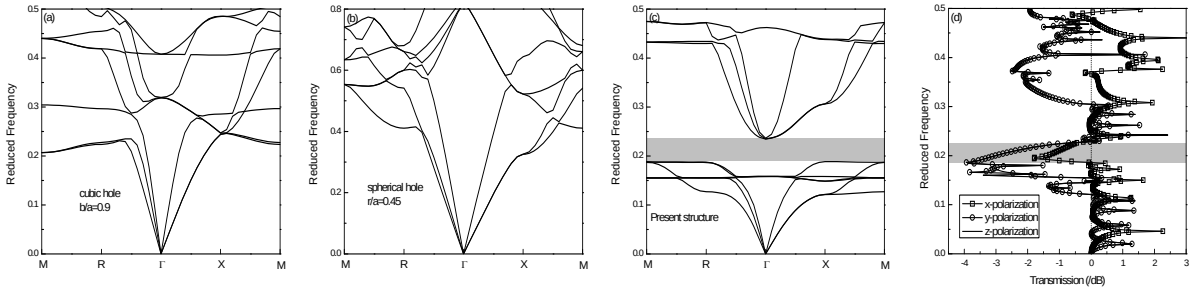

Figure.2 Band structures for phononic crystals with cubic holes (a), spherical holes (b) and present structure (c). The transmission spectra (d) of a $5a\times 1a$ system along ΓX direction are calculated to confirm the band structures shown in panel (c).

It is clear that no complete bandgap exists in the band structures for the system with either the cubic holes or the spherical holes. Still we find that no complete bandgap appears whatever the size of the cubic or the spherical holes. While for the present structure, a complete bandgap exists in the frequency range of $0.18 < \Omega < 0.23$ between the 6th and 7th bands. The bandgap width to midgap frequency ratio is 24%, which is wide enough for practical applications. To confirm the analysis of the band structures, the transmission spectra of a $5a\times 1a$ system are calculated along the ΓX direction, see Fig. 2(d). The partial bandgap (the shadowed region) in the transmission spectra is in coherent with the band structures. This shows the validity and accuracy of our theoretical predictions of the complete bandgaps.

In order to understand the origin of the bandgap, we calculate the vibration modes at the band edges of the complete bandgap. For the lower edge-mode, the lump oscillates with the connectors acting as springs, and the external frame is almost still. While for the upper edge-mode, the lump and the external frame vibrate in a reverse phase. So it is clear that the emergence of the complete bandgap is due to the local resonance of the large lump. Then spring-mass models are developed to predict the frequencies at the band edges. Generally speaking, the results obtained by the heuristic models are in agreement with the numerical results.

CONCLUSIONS

We show in this paper that by careful design of the geometry of the multiply connected holes, complete bandgap with relatively low center frequency can be obtained for three dimensional phononic crystals in a simple cubic lattice. The origin of the bandgap is due to the local resonance of the unit cell. Spring-mass models are developed to predict the boundaries of the complete bandgap. The predicted results are in general agreement with the numerical results.

References

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