WAVE PROPAGATION IN ONE-DIMENSIONAL NANOSCALED PHONONIC CRYSTALS

Ali Chen[* a)], Yuesheng Wang[*]
[*] *Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing, 100044, China*

Summary The propagation of the elastic wave in one dimensional (1D) nanoscaled phononic crystal is studied by employing the conception of the localization factor which is calculated by the transfer matrix method. The size-effect is considered by using the nonlocal elastic theory. For normal incidence the influence of the internal to external characteristic length ratio R on the cut-off frequency is calculated. The influence of R on the band structures for anti-plane and in-plane waves propagating obliquely in the systems is discussed. For in-plane waves the mode exchange phenomenon is talked about.

INTRODUCTION

Phononic crystal (PNC), which was first proposed by Kushwaha et al. [1], is a kind of composites composed of periodic arrays of two or more materials with different mass densities and elastic properties. The most important property is its band gaps which have numerous potential engineering applications such as wave detectors, filters, waveguides, transducers, etc. With the rapid development of the technology in the fields of communication information, medical engineering, etc., the size of acoustic devices is required to be smaller and smaller. For instance, the gigahertz communication generally requires the nanosized devices. The hypersonic phononic band gaps could be used to design new thermo-electric devices, acoustic-optic devices, nano electric-mechanical systems (NEMS), etc.
It is known that the size-effect will become more important and should be taken into account when a system is in the dimension of several nanometers for which the scaling law [2] does not hold. Ramprasad et al. [3] presented the phononic band structure calculations for a nanoscaled HfO2-ZrO2 multilayer stack by using the first-principles method at the atomistic level and by solving the classical elastic wave equation at the continuum level, and found that the results are quite different. In Ref. [4] the authors calculated the band structures of a nanoscaled layered PNC by using the transfer matrix method based on the nonlocal elastic (NLE) continuum theory [5] which has been used to study mechanical behaviors of nanoscaled structures. The results show that the developed NLE method is expected to overcome the limits of the classical continuum description for wave propagation in PNCs when dimensions are in nanometer-length scales. As a further discussion for Ref. [4], in this paper the band structures described by the localization factors [6] for elastic waves propagating normally and obliquely in the 1D nanoscaled layered PNCs are calculated. Both the anti-plane and in-plane waves are considered.

THE MODELS AND THE METHOD

A layered PNC consisting of layers (sub-cells) A with thickness d_1 and B with thickness d_2 alternatively as shown in Fig.1 will be studied in this paper. d_1 and d_2 are at the nanoscale.

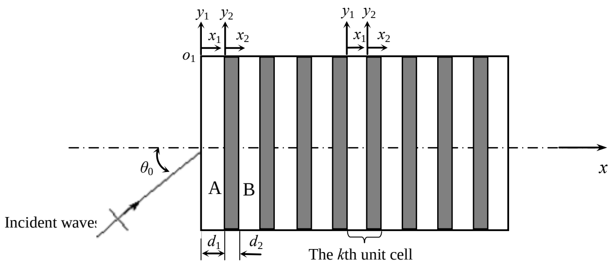

Fig. 1 Schematic diagram of a one-dimensional layered phononic crystal

The transfer matrix method based on the NLE theory

The transfer matrix method based on the NLE theory is described here. Unlike the CE theory which assumes that the stress at a point is solely dependent on the local strain field at this point, the NLE theory takes into account the effects of the long range interatomic forces and supposes that the stress-state at a point is related to the strain-state at all points of the entire body. So the differential wave motion equation of the NLE theory can be expressed as [5]:

$$(\lambda+\mu)u_{\beta,\gamma\beta}+\mu u_{\gamma,\beta\beta}=(1-\varepsilon^2\nabla^2)\rho\ddot{u}_\gamma \qquad (1)$$

where ε is the internal characteristic length. Then the transfer matrix method can be used.

Localization factor

a) Corresponding author. Email: alchen@bjtu.edu.cn.

The localization factor, similar to the Lyapunov exponent, applied to characterize the spatial evolution of a nearly periodic system is the average exponential rate of growth or decay of the wave amplitudes. Consider a 2m × 2m transfer matrix. The expression for calculating the localization factor of the system was given by Wolf [6]:

$$\gamma_m = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \ln \left\| \hat{\mathbf{v}}_{2R,m}^{(k)} \right\| \tag{2}$$

where the calculation of the vector $\hat{\mathbf{v}}_{2R,m}^{(k)}$ can be found in Ref.[7].

NUMERICAL EXAMPLES AND DISCUSSIONS

The propagations of the anti-plane and in-plane waves in the nanoscaled HfO2-ZrO2 periodic layered structures as shown in Fig.1 are studied. For saving place here we just give the gray-scale maps (as shown in Fig. 2) of the localization factors varying with the normalized frequency and $\bar{k}_y$ for anti-plane waves propagating obliquely in the 1D nanoscaled PNCs with different *R*.

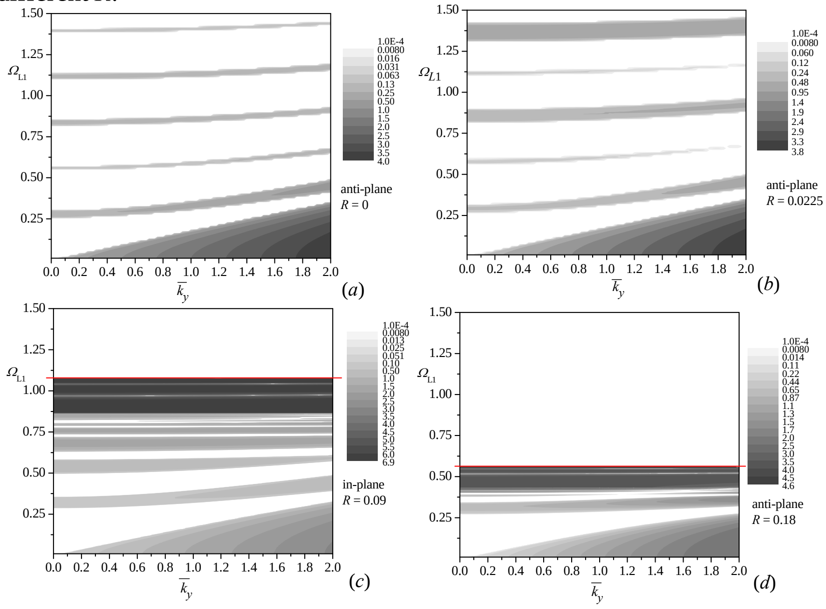

Fig. 2 The localization factors vary with the normalized frequency and $\bar{k}_y$ for anti-plane waves propagating obliquely in the 1D nanoscaled PNCs with different *R*

It can be seen that with the increase of *R* the bands and band gaps will be compressed under the cut-off frequency which was described by the red lines in Figs. 2(c) and (d). Moreover nearby the cut-off frequency the localization phenomenon becomes very strong. With the increase of *R* the band gaps become wider and the bands become slimmer. But there is only little influence of *R* on the lowest band gap.

CONCLUSIONS

The influence of R on the band structures for anti-plane and in-plane waves propagating obliquely in the systems is discussed. For in-plane waves the mode exchange phenomenon is talked about. The results show that for anti-plane waves propagating obliquely in the system, the band gaps become wider and the bands become slimmer with the increase of R. For in-plane waves propagating obliquely in the 1D systewm, most band gaps are discontinuous in which the wave mode exchange phenomenon appears. With the increase of R the band gaps in the high frequency becomes more and more and will merge into several widerand continuous ones. And there is only little influence of R on the lowest band gap.

References

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