ACOUSTOELASTIC THEORY FOR A FLUID-SATURATED POROUS MEDIA

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Summary Based on the finite deformation theory of the continuum and poroelastic theory, acoustoealstic theory for a fluid-saturated porous media (FSPM) in the natural coordinate is developed to investigate the influence of the initial effective stress and the initial pore pressure on wave velocities. Consider a small motion superimposed on a largely pre-deformation of FSPM, the subtraction of the initial-state equation of equilibrium from the final-state equation of motion and the introduction of the non-linear constitutive relation lead to equation of motion for the small dynamic motion. The consideration of the homogeneous pre-deformation gives the analytical formulation for the relation of the wave velocities with the solid-skeleton stresses and the pore pressure. An isotropic FSPM under the open-pore jacketed condition is taken as an example to show that the wave velocities of the FSPM are usually influenced by the effective stress and pore pressure. The pore pressure has no effect on the wave velocities of FSPM only when the components of the initial principal stress or strain are equal, which is consistent with the previous experimental results.

INTRODUCTION

The mechanical properties in a fluid-saturated porous media (FSPM), which is composed by the solid skeleton and the abundant connected pore space completely filled with the fluid, plays an important role in the research fields of geotechnical engineering, geophysical prospecting, acoustics, etc.. Since Biot established the poroelasticity theory[1-3], extensive researches for the properties of the FSPM have been investigated, especially for the acoustic properties. As a nonlinear acoustic properties, acoustoelastic phenomenon of the FSPM, which describes the wave-velocities change induced by the initial solid-skeleton stresses and the fluid pore-pressure, has wide application in the estimation of the crustal stresses and pore pressures. The rock experiments and the geophysical field observations have shown that acoustoelasticity in the FSPM is much greater than that of the purely elastic solid [4-7] .
Although the acoustoelastic phenomenon in the FSPM has been verified by the geophysical field observations and rock experiments, the researches in the theoretical analysis for this phenomenon are very few because of the nonlinear characteristic. In 1963, Biot adopted the nonlinear continuum theory to firstly investigate the acoustoelasticity theory for the FSPM and only present the equations of motion for the FSPM under the initial stress[8], but the influence of the initial stress on the wave velocity based on these equations of motion did not be considered until the work of Sharma in 2005 [9]. Grinfeld and Norris combined the governing equations of the nonviscous, fluid-filled poroelastic media in a Lagrangian coordinate with the compatibility conditions of acceleration waves and the nonlinear constitutive equation to derive the formulation of the stress-induced wave-velocity variation by the standard perturbation theory [10]. The above-mentioned works did not provide the explicit formulations about the influence of the effective stress and pore pressure on the wave velocities of the FSPM and explain the experimental results about the independence of the pore pressure on the wave-velocities for the constant effective confining stresses.
The aim of this paper is to develop the acoustoelasticity theory for the FSPM in the natural coordinates based on the finite deformation theory of the continuum and poroelastic theory and present the explicit formulation about the influence of the effective stresses and the fluid pore-pressure on the wave velocities. In order to simplify the analysis, it is assumed that there is no dissipation induced by the fluid-solid relative motion in the FSPM.

WAVE PROPAGATION IN FINITE DEFORMATION NON- DISSIPATIVE FSPM

Combining the Biot theory with the nonlinear continuum theory, a small disturbance superimposed on a static pre-deformed FSPM is considered to develop the acoustoelasticity theory of a homogeneous and hyper-elastic FSPM in the natural coordinates. Here it is assumed that the scale of wave length for the small disturbance is much greater than that of the pore size of the FSPM and the relative motion of the fluid in the pores is of the Poiseuille type.
There are three different configurations for the small motion superimposed on a large deformed FSPM, including the natural configuration, the initial configuration, and the final configuration. The FSPM in the original state free of stress and strain is called the natural configuration. The second is initial configuration for a static deformed FSPM. When a small motion is superposed on the FSPM in the initial state, the FSPM is further deformed to the final configuration.
The subtraction of the initial-state equations of equilibrium from final-state equations of motion and the introduction of the non-linear constitutive relations of the FSPM lead to equations of motion for the small dynamic motion

$$\left.\begin{aligned} &\frac{\partial}{\partial \xi_\beta}\left[\bar{T}_{\alpha\beta} + \bar{T}^i_{\beta\gamma}\frac{\partial u_\alpha}{\partial \xi_\gamma} + \bar{T}_{\beta\gamma}\frac{\partial u^i_\alpha}{\partial \xi_\gamma}\right] = \rho^0_{11}\frac{\partial^2 u_\alpha}{\partial t^2} + \rho^0_{12}\frac{\partial^2 U_\alpha}{\partial t^2} \\ &\frac{\partial}{\partial \xi_\beta}\left[s\left(\delta_{\alpha\beta} + \frac{\partial u^i_\alpha}{\partial \xi_\beta}\right) + s^i\frac{\partial u_\alpha}{\partial \xi_\beta}\right] = \rho^0_{12}\frac{\partial^2 u_\alpha}{\partial t^2} + \rho^0_{22}\frac{\partial^2 U_\alpha}{\partial t^2} \end{aligned}\right\} \tag{1}$$

Consideration of the plane wave form of the small wave motion for the homogeneous FSPM yields the characteristic equation about the wave velocity c in the natural coordinate

$$\left|\begin{bmatrix} A_{\alpha\beta\gamma\delta}v_\beta v_\delta & \Phi_{\alpha\beta}v_\beta v_\gamma \\ B_{\alpha\beta\gamma\delta}v_\beta v_\delta & Z_{\alpha\beta}v_\beta v_\gamma \end{bmatrix} - c^2\begin{bmatrix} \rho^0_{11}\delta_{\alpha\gamma} & \rho^0_{12}\delta_{\alpha\gamma} \\ \rho^0_{12}\delta_{\alpha\gamma} & \rho^0_{22}\delta_{\alpha\gamma} \end{bmatrix}\right| = 0\,. \tag{2}$$

There are four nonzero wave velocities corresponding to the slow and fast longitudinal waves and the slow and fast shear waves.

ACOUSTOELASTICITY OF ISOTROPY FSPM

Let the coordinate axes coincide with the principal axes of an isotropic FSPM in the natural coordinate, the consideration of a plane wave propagating in the direction of the ξ_3 axis provides the implicit formulation for the two shear-wave velocities as

$$\left.\begin{aligned} c_3^2 &= c_0^2\left(1+\left(\bar{T}_{33}^i+\left(2c_{44}+c_{551}\right)e_{11}^i+c_{552}e_{22}^i+c_{551}e_{33}^i-\alpha_1 s^i\right)/c_{44}\right) \\ c_4^2 &= c_0^2\left(1+\left(\bar{T}_{33}^i+c_{552}e_{11}^i+\left(2c_{44}+c_{551}\right)e_{22}^i+c_{551}e_{33}^i-\alpha_1 s^i\right)/c_{44}\right) \end{aligned}\right\} \tag{3}$$

RESULTS AND DISCUSSIONS

It is shown in Eq. (3) that wave velocities of the pre-deformed FSPM are determined by not only the second-order and third-order elastic constants of the FSPM, but also the initial solid-skeleton and fluid stresses. Furthermore, the initial stresses and strains of FSPM are greatly affected by the loading conditions. Therefore, the consideration of the open-pore jacketed condition for the FSPM of the applied confining pressures and the constant pore pressure yields the fast and slow shear-wave velocities as

$$\left.\begin{aligned} c_3{}^2 &= c_0^2\left(1+A_2\bar{\sigma}_1+B_2\bar{\sigma}_2+A_2\bar{\sigma}_3-\left(\left(2A_2+B_2\right)\alpha+C_2 n\right)p_f\right) \\ c_4{}^2 &= c_0^2\left(1+B_2\bar{\sigma}_1+A_2\bar{\sigma}_2+A_2\bar{\sigma}_3-\left(\left(2A_2+B_2\right)\alpha+C_2 n\right)p_f\right) \end{aligned}\right\}, \tag{4}$$

where $\bar{\sigma}_i = \sigma_i + \alpha p_f \cdot \alpha$ is the effective stress coefficient. The ratio of the coefficients of the pore pressure and the effective principal stresses usually doesn't approach zero, which means that the pore pressure has great influence on the wave velocities. However, when the components of the applied initial principal stress are equal, the ratio of the coefficients of the pore pressure and the effective principal stresses approximates zero, which represents the independence of wave velocities on the pore pressure for the equal effective stresses. Because the tectonic principal stresses in the shallow crust are usually not equal, the influence of pore pressure on the wave velocity is larger so that the field observation data of wave velocity could be used to estimate the pore pressure of the crust. But the deep crust is subjected to hydrostatic pressure, where wave velocity is only determined by the effective stresses. Therefore, we must apply the wave velocities to the estimation of the abnormal pore pressure in the deep crust.

CONCLUSIONS

In conclusion, acoustoealstic theory for the FSPM considering the combination of the finite-deformation theory of the continuum and poroelastic theory has been developed to show that the wave velocities for the isotropic pre-deformed FSPM under the close-pore jacketed condition are usually influenced by the effective stresses and the pore pressure. The pore pressure has no effect on the wave velocities of the FSPM only when the components of the initial principal stress are equal.

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