OPTIMIZATION OF FEEDBACK CONTROL OF FLOW OVER A CIRCULAR CYLINDER

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Summary The purpose of present study is to perform a feedback gain optimization of the proportional-integral-differential (PID) control for flow over a circular cylinder. The Reynolds number considered are $Re = 60$ and 100. A sensor is located along the centerline of the wake and actuations (blowing and suction) are given from the upper and lower slots on the cylinder surface. Given each sensing location, the optimal feedback gains in the sense of minimizing the velocity fluctuations at sensor location are obtained by iterative feedback tuning method. The optimized PID control successfully reduces the velocity fluctuations at the sensing location and attenuates (or annihilates) vortex shedding in the wake.

INTRODUCTION

To suppress vortex shedding behind a bluff body, many different feedback control methods have been studied so far [1]. Recently, Son *et al.* [2] performed a proportional-integral-differential (PID) control of flow over a circular cylinder and successfully suppressed the vortex shedding. However, to obtain a successful control result, they performed many case studies about three parameters of PID control, α, β and γ, which were the proportional, integral and differential feedback gains, respectively (Fig. 1 (*a*)). Also, the parameter values obtained from the manual tuning were not optimal for the given feedback control system. Therefore, to avoid the manual tuning in the process of feedback control and to obtain optimal values of the feedback gains, we perform a direct feedback tuning method, so called iterative feedback tuning method.

The PID control is given as $\psi(t) = \alpha v_s(x_s, t) + \beta \int_0^t v_s(x_s, \tau) d\tau + \gamma \frac{d}{dt} v_s(x_s, t)$, where ψ is the radial actuation velocity (blowing/suction) from the upper and lower slots on the cylinder surface, and v_s is the measured velocity at the sensing position x_s (centerline of the cylinder wake). In the present study, we set the slot locations at $100° \leq \theta \leq 120°$ just before the separation points for $Re = 60$ and 100 (Fig. 1 (*c*)). Various sensing locations are tested: $x_s/d = 1 \sim 4$ by increments of 0.5. The target of the PID control is to reduce the error that is the sensing velocity at the centerline location of the cylinder wake. Therefore, a successful control should reduce the root-mean-square (r.m.s.) transverse velocity fluctuations at the sensing location, which in turn possibly attenuates the vortex shedding and reduces the mean drag and lift fluctuations.

ITERATIVE FEEDBACK TUNING METHOD

Iterative feedback tuning (IFT) method is a typical model free gain optimization method proposed by Hjalmarsson *et al.* [3]. One of the most important advantages of the IFT method is that it can optimize the controller gains for a given plant although the transfer function of the given plant is unknown. Especially, a plant under consideration is complex or even non-linear, the IFT method may optimize the controller gains and successfully enhance the performance of the controller. A fluid system governed by the Navier-Stokes equations is a typical non-linear system, and it is difficult to find a transfer function of this control system. Therefore, the IFT method is a proper option to optimize the present control system, i.e., the control of flow over a circular cylinder.

To perform the ITF method, we choose a suitable cost function J that we want to minimize. Obviously, if we choose an improper cost function, it is not possible to obtain good control performance. In the present study, the process error is the sensing velocity itself. Therefore, we define the cost function using the sensing velocity v_s as $J(\rho) = \frac{1}{T_f} \int_{T_f/2}^{T_f} \left\{ v_s^2(\tau, \rho) + \lambda^2 \psi^2(\tau, \rho) \right\} d\tau$, where $\rho = [\alpha, \beta, \gamma]^T$ is the PID controller gain vector. T_f is the time interval for the sensing and is sufficiently long time at which the plant shows statistically steady state at the given gain vector ρ. λ^2 is the weighting constant of the cost function. In the present study, we choose $T_f = 100$ and $\lambda^2 = 0$. The initial gain vector is $[0, 0, 0]^T$ that corresponds to the case of no control. To minimize the cost function $J(\rho)$, we update the gain vector ρ: $\rho(k+1) = \rho(k) - \kappa(k) R(k)^{-1} \frac{\partial}{\partial \rho} J(\rho(k))$. Here, $R(k)$ is an appropriate positive definite matrix, typically a Gauss-Newton approximation of the Hessian of $J(\rho(k))$, $\kappa(k)$ is a positive real scalar that determines the step size, and k is the iteration index. We determine the step size $\kappa(k)$ by the golden section method (GSM). The gradient of the cost function is expressed as $\frac{\partial}{\partial \rho} J(\rho(k)) = \frac{2}{T_f} \int_{T_f/2}^{T_f} v_s(\tau, \rho(k)) \frac{\partial}{\partial \rho} v_s(\tau, \rho(k)) d\tau$. To obtain $\frac{\partial}{\partial \rho} J(\rho(k))$, we perform one additional parallel loop of the control system that the reference input is replaced with $v_s(t)$ (Fig. 1 (*b*)). Detailed procedures for the IFT method are available in [3].

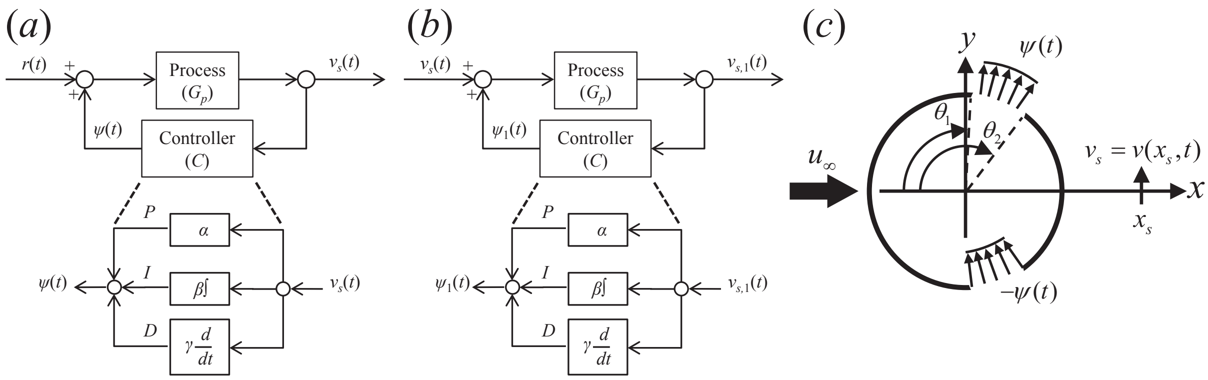

Figure 1. Schematic of the control system: (*a*) block diagram of the PID control system; (*b*) block diagram of the parallel control system for gradient of cost function estimation; (*c*) coordinates, sensing and actuation for flow over a circular cylinder.

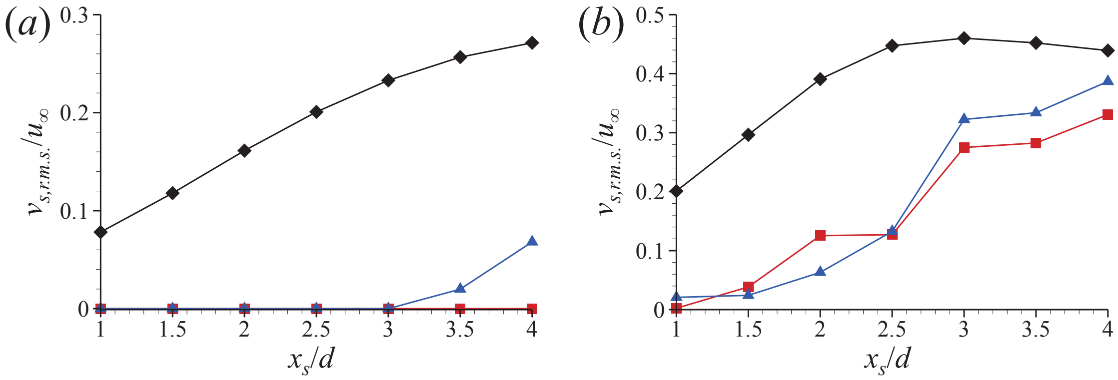

Figure 2. Controlled sensing velocity fluctuations vs. sensing position: (*a*) *Re* = 60; (*b*) 100. Squares, present; triangles, Son *et al.* [2]; diamonds, no control.

CONTROL RESULTS

We optimize the PID control gains for flow over a circular cylinder with the IFT method. As a result, PID-IFT successfully finds a local optimum of the control gains at all the sensing positions. Figure 2 shows rms of controlled sensing velocity fluctuations as a function of the sensing position. In the case of $Re = 60$ and $x_s/d = 2.0$, optimal gains are obtained by three gradient estimations and twenty function evaluations during the GSM, and they are $[-0.0365, 0.3228, -0.4007]^T$. In the case of $Re = 60$, the present PID-IFT successfully suppresses the vortex shedding at the sensing positions from $1d$ to $4d$. In the case of $x_s/d = 3.5$ and 4.0, present results show better control performance than those of the manual tuning by Son *et al.* [1]. In the case of $Re = 100$, the control performance is not better than that of $Re = 60$, but it is still very good with the optimized feedback gains.

CONCLUSIONS

In the present study, we performed the feedback gain optimization of the PID control. We applied the PID control to flow over a circular cylinder at $Re = 60$ and 100 for suppression of vortex shedding in the wake. Given each sensing location, the optimal feedback gains of minimizing the sensing velocity fluctuations were obtained by the iterative feedback tuning (IFT) procedure. The optimized PID-IFT control successfully reduced the velocity fluctuations at sensing locations and attenuated the vortex shedding in the wake.

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References

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