ATTENUATION OF SEA WAVES BY A VEGETATION CANOPY

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Summary

Motivated by environmental concern, the dynamics of steady currents through submerged vegetation has been studied by experiments and numerical modeling [1][2]. For transient waves or tides, past efforts have mainly focussed on large-scale simulation based on empirical estimates of drag coefficients etc. To examine the damping of long waves through emergent vegetation in shallow water, Mei et al. (2011) [3] applied the theory of homogenization to deduce the effective equation governing the large-scale physics from micro-scale analysis. Turbulene is modeled by a constant eddy viscosity measured from past experiments for steady flows. In this work we extend their theory to shorter wind waves propagating in water of intermediate depth over a canopy of small height. Vegetation is represented by periodically spaced vertical cylinders. The macro-scale flow through the canopy is shown to resemble transient seepage in porous media. The permeability is computed as a function of frequency from only a canonical problem for single cell on the micro-scale. Effects of canopy porosity and height on the spatial attenuation of waves will be compared with experiments.

THE PROBLEM AND APPROACH

Stimulated by the retardation of tsunami by coral reefs, mangroves and other coastal forests, Mei et al. [3] have described a theory for transient long waves propagating through an emergent vegetation in shallow water. To model turbulence among trees a simple model of constant eddy viscosity μ_e is adopted. The values of μ_e for different tree density are taken from existing experiments for steady flows. The asymptotic theory of homogenization is used to derive equations governing the large-scale by analyzing the dynamics on the small scale. Numerical computation is limited to the solution of a canonical cell problem for a unit cell. Predictions of wave evolution for both periodic and pulse-like waves have been confirmed by laboratory experiments.

In this work the homogenization analysis is applied to a submerged canopy in a sea of depth h comparable to the wave length $2\pi/k$. The canopy height H is assumed to be much smaller than the sea depth h. Only small amplitude waves are considered. The macro-scale flow above $(0 < Z < h)$ and outside the canopy is assumed to be governed by the velocity potential ϕ which satisfies Laplace's equation and the the familiar free-surface conditions. To examine the flow inside the canopy $(-H < Z < 0)$, fast and slow coordinates are introduced in the horizontal directions : the fast coordinates $(\vec{x}, z)$ are for the micro-scale of tree spacing $O(\ell)$ and (X_i, Z) for the macro-scale of the wavelength $O(1/k)$, with $k\ell \ll 1$. By the two-scale analysis it is shown that the cell-averaged horizontal velocity and the vertical velocity at the canopy top are related to the averaged pressure by

$$\left\langle u_i(\vec{X})\right\rangle = -\left\langle K_{ij}\right\rangle \frac{\partial \left\langle p\right\rangle}{\partial X_j}, \quad \left\langle w(\vec{X}, Z=0)\right\rangle = H\frac{\partial}{\partial X_i}\left(\left\langle K_{ij}\right\rangle \frac{\partial \left\langle p\right\rangle}{\partial X_j}\right) \tag{0.1}$$

The permeability tensor $\langle K_{ij}\rangle$ is the cell average of $K_{ij}(\vec{x})$ which is obtained from the finite-element solution of a two dimensional boundary value problem in a unit cell. For a square array of circular cylinders, sample variations of K_{11} inside a cell is shown in Figure 1. The diagonal elements $K_{12}(\vec{x}) = K_{21}(\vec{x})$ give zero cell averages and are not shown.

The macroscale flows in and outside the canopy are solved together by matching the dynamic pressure and the vertical velocity at the top of canopy $Z = 0$.

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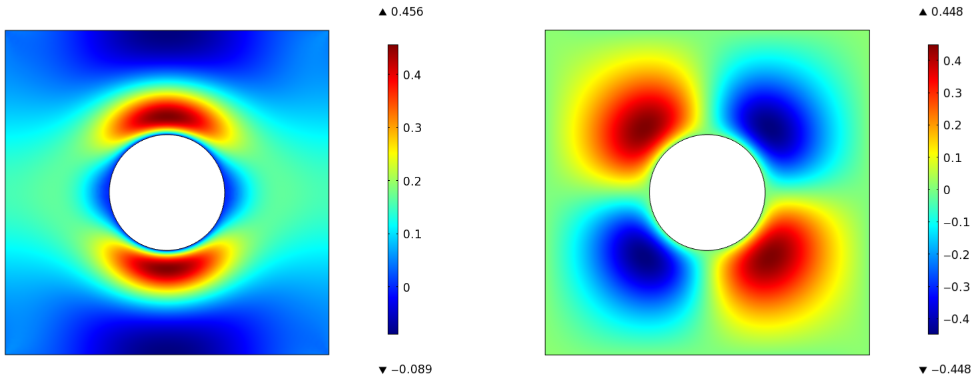

Figure 1. Spatial distribution of $\widetilde{K}_{11}$ (normalized K_{11}) in a unit cell problem with a circular cylinder inside a square. Left panel: real part of $\widetilde{K}_{11}$, Right panel: imaginary part of $\widetilde{K}_{11}$. Porosity : $n = 0.9$. Bottom friction is ignored.

SAMPLE RESULTS

As an application the spatial attenuation of a a long-crested wave train across a very wide canopy is found to be

$$\frac{A(kX)}{A(0)} = \exp\left(-kH\ kX\ \frac{\widetilde{K}}{n}\ \frac{1}{\sinh 2kh}\frac{C}{C_g}\right) \tag{0.2}$$

where $\widetilde{K} = \rho\omega K$ is the normalized permeability, n is the canopy porosity, and C and C_g are the phase and group velocities respectively. Sample results for two porosities and three kh are shown in Fig.2. It can be seen that the attenuation ratio $|A(kX)/A(0)|$ depends not only on the nondimensional permeability $\widetilde{K}$, but also the depth-to-wavelength ratio kh. For the same canopy geometry and water depth, it is found that waves of some intermediate length has the fastest attenuation, as can be seen from the circles marking the amplitudes at the same station $X = 40\pi h$. Computations show that the absolute value of $\widetilde{K}$ increases with kh. While $C/2C_g$ also increases from 1/2 to 1, the factor $1/\sin 2kh$ decreases with kh exponentially. As a consequence the maximum decay rate at a given station is attained by waves of some intermediate length.

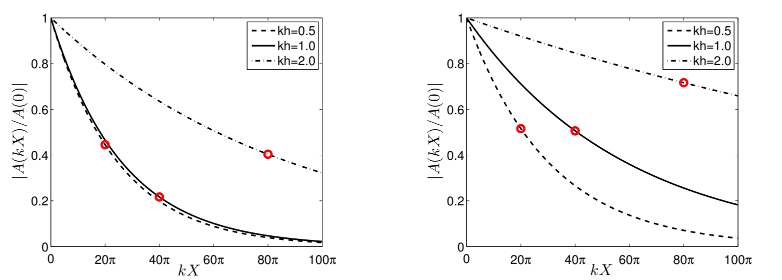

Figure 2. Variation of wave amplitude $|A(X)/A(0)|$ versus nondimensional distance kX' for depth-to-wave length ratio $kh = 0.5$, 1.0 and 2.0. Left panel: porosity n=0.8, Right: n=0.9. Input parameters are $h = 10$m, $H = 1$m, $\ell = 1$m, $A/h = 0.05$.

Theoretical predictions and laboratory experiments will be presented for wave scattering by a canopy of finite extent.

References

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