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SIMULATION-BASED STUDY OF WAVE–TURBULENCE INTERACTION

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Summary Direct numerical simulation is performed to study interaction of sub-surface turbulence with a deformable free surface and a surface wave. Two types of surface motion induced by the turbulence are identified, propagating surface waves and turbulence-induced surface roughness. The magnitude of turbulence vorticity is found to be dominated by the wave stretching effect. Vertical vorticity is observed to be turned to the wave-propagation direction by the accumulative wave turning effect.

NUMERICAL METHOD

This study aims at using direct numerical simulation (DNS) to improve our understanding of the interaction of sub-surface turbulence with deformable free surfaces and surface waves, which is important to many environmental and engineering applications. In the simulation, the flow is in the frame (x, y, z), where x, y, and z point to the streamwise, spanwise, and vertical directions, respectively. The fluid motion is governed by the Navier–Stokes equations and the continuity equation with fully nonlinear kinematic and dynamic boundary conditions at the free surface. The simulation is performed on a boundary-fitted grid that follows the motion of the free surface. For spatial discretization, a Fourier-based pseudospectral method is used in the horizontal directions, and a second-order finite-difference method is used in the vertical direction. For time integration, a semi-implicit second-order fractional-step method is used. The details of the numerical scheme are given in [1, 2, 4].

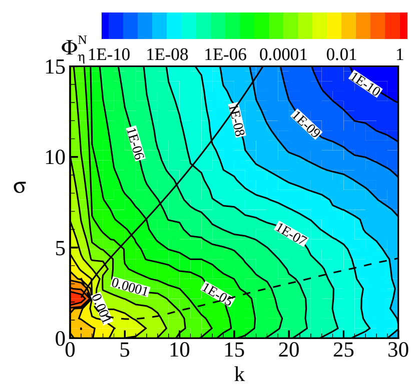

Figure 1: Normalized frequency–wavenumber spectrum of the surface elevation. ——, wave dispersion relationship, σ_w; – – –, characteristic frequency, σ_t.

In this study, normalization is performed based on a characteristic length scale, L, which is $1/(2\pi)$ of the horizontal computational domain size, a characteristic velocity scale, U, which is about 10 times of the turbulence velocity fluctuation, and the density of water, ρ. We set the dimensionless computational domain size as $L_x \times L_y \times L_z = 2\pi \times 2\pi \times 5\pi$. We use an evenly-distributed grid with 128 points in the horizontal directions and a 348-point grid, which is clustered towards the free surface, in the vertical direction. In the viscous layer of the free surface, there are at least 7 points to ensure that the viscous effect is captured accurately. The turbulence field considered here is studied in detail in [2]. Here, we summarize the non-dimensional key properties. Near the free surface, the turbulence kinetic energy $q \equiv 3(u^{rms})^2/2 = 0.0122$, the integral length scale $L_\infty = 0.69$, the Taylor-scale Reynolds number $Re_\lambda = 30$, and the turbulence dissipation rate $\epsilon = 0.001$. Here, the superscript 'rms' denotes the root-mean-square value. For the surface wave, we set its amplitude $a = 0.1$, wavenumber $k = 1$, and frequency $\sigma = 10$. The turbulence-to-wave length ratio $L_\infty/(2\pi/k) = 0.1 \ll 1$. The turbulence-to-wave time ratio $ak\sigma q/\epsilon = 11.5 \gg 1$. Therefore, the case considered here belongs to the rapid distortion case [3].

RESULTS

Effect of turbulence on surface signatures

We first study the effect of turbulence on surface signatures. In this part of study, there is no dominant surface wave and the surface motion is caused by the underlying turbulence. To understand the features of surface motion, we study the normalized frequency–wavenumber spectrum of the surface elevation, η, defined as

$$\Phi_\eta^N\left(\left|\vec{k}\right|, \sigma\right) = \frac{1}{(2\pi)^3 \cdot (\eta^{rms})^2} \int_T \int_S \overline{\eta\left(\vec{x}, t\right) \eta\left(\vec{x}+\vec{r}, t+\tau\right)} \cdot \mathrm{e}^{-\mathrm{i}\left(\vec{k}\cdot\vec{r}+\sigma\tau\right)} \,\mathrm{d}\vec{r}\,\mathrm{d}\tau. \tag{1}$$

Here, T is the sampling duration; S denotes the horizontal plane; the overline $\overline{(\cdot)}$ denotes averaging.
Fig. 1 plots the contours of Φ_η^N for a case with $Fr^2 = 0.2$ and $We^{-1} = 0.025$, where the Froude number $Fr \equiv U/\sqrt{Lg}$ and the Weber number $We \equiv \rho U^2 L/\gamma$ with g being the gravitational acceleration and γ being the surface tension coefficient. As shown, the energy mainly concentrates along two ridges, one with relatively high frequency and the other with lower frequency. The two ridges correspond to two types of surface motion. One is propagating surface waves and the other is turbulence-induced surface roughness.
To show the motion of surface wave and turbulence-induced roughness, we plot in fig. 1 the surface wave dispersion relationship, $\sigma_w = \sqrt{k/Fr^2 + k^3 \cdot We^{-1}}$ and the characteristic frequency defined as $\sigma_t = \sqrt{\Psi_\eta(k)/\Psi_{\eta_t}(k)}$ with

$$\Psi_q\left(\left|\vec{k}\right|\right) = \frac{1}{(2\pi)^2} \int_S \overline{q\left(\vec{x}, t\right) q\left(\vec{x}+\vec{r}, t\right)} \cdot \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}} \,\mathrm{d}\vec{r}. \tag{2}$$

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The curve of σ_w fits well with the ridge corresponding to propagating waves, and σ_t fits well with the ridge corresponding to turbulence-induced roughness. The surface wave motion is mainly at large scales. The turbulence-induced motion occurs at all the scales because of the turbulence at the corresponding scales. At large scales, the difference between σ_t and σ_w is small, indicating that surface waves may be excited by the turbulence motion. At small scales, the difference becomes large and as a result, turbulence dissipates surface waves rather than generating them.

Effect of surface wave on turbulence vorticity

We next study the effect of surface wave on the underlying turbulence with a focus on vorticity. In this part of study, a dominant progressive wave propagates above the turbulence field. By observing instantaneous turbulence vorticity field (results not shown here), we found that turbulence vorticity components in the streamwise and vertical directions are strongly affected by the wave strain field. Our statistical analysis confirms this observation. By examining the vorticity evolution equations, we found that vorticity stretching terms ($\omega_x \cdot \partial u^w/\partial x$ and $\omega_z \cdot \partial w^w/\partial z$) and vorticity turning term ($\omega_z \cdot \partial u^w/\partial z$) shown in fig. 2(a–c) are important to the evolution of turbulence vorticity. Here, the superscript 'w' denotes wave motion.

For the streamwise vorticity, fig. 2(a) plots the contours of $\langle|\omega_x| \cdot \partial u^w/\partial x\rangle$. Here, $\langle\cdot\rangle$ denotes the wave-phase-based average. Note that in the analysis, the absolute value of vorticity is used, because the mechanism is the same regardless of the sign of vorticity. When the forward slope passes, $\langle|\omega_x| \cdot \partial u^w/\partial x\rangle$ is negative, indicating that ω_x is compressed and reaches its minimum magnitude under the wave crest. The opposite happens when the backward slope passes.

For the vertical vorticity, fig. 2(b) plots the contours of $\langle\omega_z \cdot \partial w^w/\partial z\rangle$. The $\langle\omega_z \cdot \partial w^w/\partial z\rangle$ is positive under the forward slope, indicating that ω_z is stretched and reaches its maximum magnitude under the wave crest. The opposite process happens under the backward slope. As a result, the wave stretching and compression effect leads to a wave-phase dependent distribution of turbulence vorticity.

Besides the stretching effect, vorticity turning effect is also found to be important. Fig. 2(c) plots the contours of $\langle\omega_z \cdot \partial u^w/\partial z\rangle$, which are positive under the wave crest and negative under the wave trough. This distribution indicates that the vertical vorticity is turned to the clockwise direction under the wave crest and to the anti-clockwise direction under the wave trough. Comparison of the magnitude of $\langle\omega_z \cdot \partial u^w/\partial z\rangle$ between under the wave crest and under the wave trough shows that the turning to the clockwise direction dominates. Therefore, the net effect is to turn the vertical vorticity clockwise to the wave propagation direction. This result is consistent with previous theoretical analysis [3], which shows that the accumulative effect of surface wave turns the vertical vorticity to the streamwise direction, leading to the initial seeds of small scale Langmuir cells.

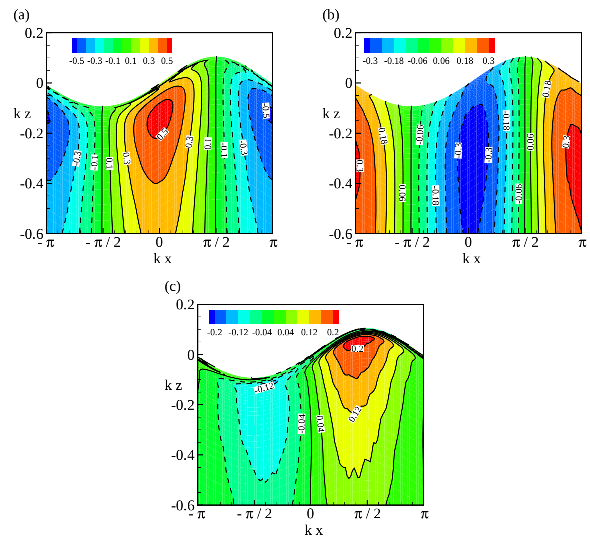

Figure 2: Contours of (a) $\langle|\omega_x| \cdot \partial u^w/\partial x\rangle$, (b) $\langle|\omega_z| \cdot \partial w^w/\partial z\rangle$, and (c) $\langle|\omega_x| \cdot \partial w^w/\partial x\rangle$. The results are normalized by $\omega'^{rms} \cdot ak\sigma$. The wave propagates from left to right.

CONCLUSIONS

In this study, we have used direct numerical simulation to study the interaction of turbulence with a deformable free surface and a surface wave. Our study on the effect of turbulence on surface signatures shows that turbulence generates two types of surface motion, propagating surface waves and turbulence-induced surface roughness. The wave motion is mainly at large scales. The turbulence-induced motion is at all the scales and is described well by a characteristic frequency. Our study on the effect of the surface wave on the turbulence voriticity reveals features of turbulence vortical structures under the distortion of the progressive wave. Turbulence vorticity becomes strongly wave-phase dependent due to the stretching and compression by the wave motion. The vertical vorticity is turned to the streamwise direction by the accumulative effect of surface wave.

References

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