NUMERICAL STUDY OF PERIOD-TRIPLING STATE IN FARADAY WAVES

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Summary We study Faraday waves in the two dimensions by a numerical simulation with the phase-field modeling of binary fluids. Validity of the simulation is checked against the linear stability analysis. Then the period-tripling state of Faraday waves observed in some experiments is reproduced. From the phase diagram, we find that the parameter region of the period-tripling states is the boundary which separates the regions of the basic period states and the quasi-periodic states.

INTRODUCTION

Although having a history of about two hundred years, Faraday waves are still actively studied. In particular Faraday waves in strongly nonlinear regime beyond the pattern formation, such as oscillons, are of great interest. In numerical studies, a binary-fluid simulation (solving dynamics of the top and the bottom fluids simultaneously) of Faraday waves in the three dimensions is performed for the first time by Périnet et al.[1] in 2009. Their simulation reproduces the hexagonal pattern, where quantitative agreement with experimental data is obtained. We believe that such a simulation in a binary-fluid setting plays an important role in understanding physical mechanisms of Faraday waves in the strongly nonlinear regimes.

In this study, we simulate Faraday waves adopting the phase-field model [2], which is one of the standard interface models in simulations of the multiphase flows. A reason of the phase-field model is its ability to handle an overturning interface seen in many experiments of strongly nonlinear phenomena. The interface model used in Périent et al.[1] is not capable to deal with such situations. We first validate the phase-field simulation by comparing with the linear stability analysis [3]. Then we study the period-tripling state of Faraday waves observed in quasi-two dimensional experiments[4] and [5], where the period of the standing wave becomes three times the basic period of the Faraday waves.

EQUATIONS OF THE PHASE-FIELD MODEL FOR THE BINARY FLUID

In this section, we describe the governing equations in this simulation, which are the imcompressive Navier-Stokes equations and the Cahn-Hilliard equation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \rho D_t \boldsymbol{u} = -\nabla p + \rho \boldsymbol{G} + \nabla \cdot \eta (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) + \boldsymbol{s}, \tag{1}$$

$$D_t \phi = \kappa \nabla^2 \mu, \quad \mu = \frac{2\sqrt{2}}{3} \frac{\sigma}{\epsilon} \left[-\epsilon^2 \nabla^2 \phi + \phi^3 - \phi \right], \tag{2}$$

where $\boldsymbol{G} = (-g + A\cos(\omega t))\boldsymbol{e}_z$ and $\boldsymbol{e}_z = (0, 1)$ in two dimensions and $g,\ A,\ \omega,\ \rho,\ \eta,\ \boldsymbol{s}$ are the gravitational acceleration, the vibration amplitude, its frequency, the density, the viscosity and the surface force. The field value ϕ, which identifies either the fluid 1 or the fluid 2, is introduced. Namely, $\phi = 1$ corresponds to the fluid 1, $-1 < \phi < 1$ corresponds to the interface and $\phi = -1$ corresponds to the fluid 2. In Eq.(2), $\kappa,\ \mu,\ \sigma,\ \epsilon$ are the mobility, the chemical potential, the surface tension coefficient and the width of the diffuse interface. In this model, ρ and η vary continuously as ϕ at the interface between two fluids. The surface force term $\boldsymbol{s}$ in the imcompressible Navier-Stokes equations (1) becomes $-\phi\nabla\mu$.

The boundary conditions of the computation are periodic in the horizontal direction and no-slip on the top and bottom rigid walls. The more detailed information is in [3]. In this direct numerical simulation of two fluids, the dynamics of vorticity and velocity which is difficult to measure in experiments are able to be calculated and the effect of the fluid-fluid interaction is able to be studied.

RESULTS

In this section, the numerical data are compared with the linear stability analysis and the period-tripling state is simulated.

Comparison with the linear analysis

In the same way as Périnet et al.[1], the numerical data using the phase-field model is compared with the linear stability analysis in the following two ways. First, we compared the critical vibration amplitudes A_c, where a perturbation is neither growing nor damping, determined from the numerical simulations with the phase-field model and the linear stability analysis. Second, the time variation of the interface in the simulation is compared with that calculated from the Floquet analysis. The critical acceleration A_c and the time variation of the neutrally stable mode calculated with the phase-field simulation of the small initial wave amplitude agree well with the results of the linear analysis. This agreement is obtained for several perturbation wavenumbers in both the harmonic and subharmonic branches. The simulation is hence validated in the linear region.

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The period tripling state

We focus on the period-tripling state which is observed in quasi-two dimensional experiments by Jiang et al.[4] and others. In this state, the period of standing waves is three times of the basic period which is $2 \times 2\pi/\omega$ in Faraday waves. The mechanism of this phenomenon is not understood well[6]. Because of limitation of our numerics, the parameters of our simulation here are different from the experiments in [4]. Nevertheless, the period-tripling state is observed. Indeed the interface variation of numerical results results shows threefold basic period in Figure 1(a). Moreover, the interface profiles of each peak are different from each other in Figure 1(b)-(d). The phase diagram obtained by the simulation is shown in Figure 2(a). The value of $p = \{2\Omega(k)/\omega\}^2$ corresponds to the dimensionless detuning parameter, where $\Omega^2(k) = kg\{\rho_{\rm dif}/\rho_{\rm sum} + \sigma k^2/(g\rho_{\rm sum})\}$. Here, $k, \rho_{\rm sum}, \rho_{\rm dif}$ are the wavenumber, the sum of two fluid densities and the difference of two fluid densities. The value of $q = (2kA\rho_{\rm dif})/(\rho_{\rm sum}\omega^2)$ corresponds to the dimensionless intensity of forcing. Only at the circular points in Fig2(a), the period-tripling states are observed. At the square points, the basic period states as in Fig.2(b) are observed. At the cross points, the behavior of the interface becomes quasi-periodic as in Fig.2(c). This diagram implies that the period-tripling region separates the parameter regions of basic states and the quasi-periodical states. There are two main differences between the simulation and the experiment by Jiang et al.[4]: the one is the interface profiles; the other one is the parameter region where the period-tripling state is observed. In the experiment, the profiles do not become the plumes as Fig.1(b) and the parameter region of the period-tripling state is much broader. Nevertheless, in our opinion, it is important that the simulation reproduces the period of the interface oscillation becoming three times of the basic period. Since the boundary conditions of this simulation are simpler than experimental boundary conditions, this makes it easier to analyze the mechanism of the period becoming threefold.

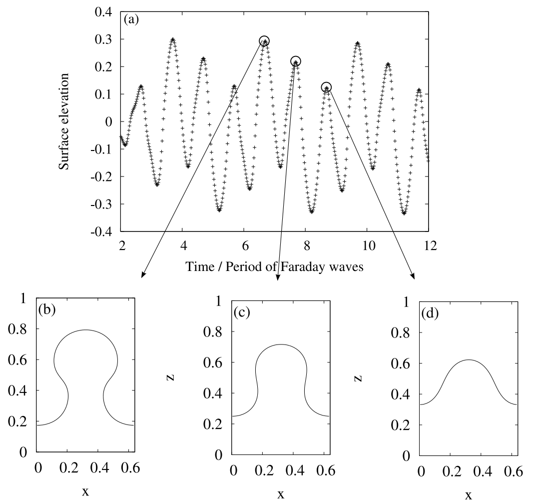

Figure 1. (a)The time variation of the interface at period tripling state: surface elevation is nondimensionized by length of x. (b)(c)(d) The interface profiles of each peak.

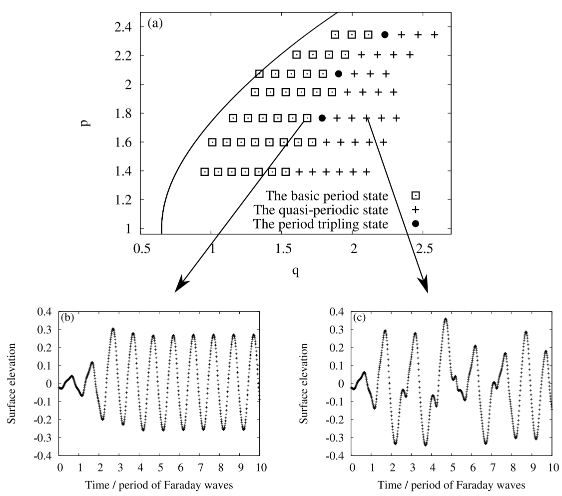

Figure 2. (a)The phase diagram. The curve corresponds to the critical line calculated from the linear stability analysis. (b)(c)The typical time variations of the interface in the basic period state and the quasi-periodic state.

CONCLUDING REMARKS

Numerical simulations of two-dimensional Faraday waves with the phase-field model in a binary fluid system are executed. Having validated the simulation by comparing with the linear stability analysis, we simulate the period-tripling state in the nonlinear regime. The result suggests that the state is observed in a horizontally periodic domain and that it forms a boundary between the basic period and the quasi-periodic states. We are now trying to find out the mechanism of period-tripling state. The findings of our simulation are useful inputs for this. Besides, since an extension of the simulation with the phase-field model to the three dimensions is straightforward in principle, we will simulate Faraday waves with the phase-field model in the three dimensions.

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