ON THE DAMPING OF ACOUSTIC WAVE PROPAGATION IN FULLY DEVELOPED TURBULENT PIPE FLOW

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Summary A frequency dependent eddy-viscosity model accounting for the nonequilibrium effects in sound-turbulence interaction is proposed which is used to predict the damping of acoustic wave propagation in low Mach number turbulent pipe flow. The simulated results show that this new model gives better agreement with the experimental data, compared with the Quasi-laminar model and the Newtonian eddy model.

INTRODUCTION

Sound-turbulence interaction in fully developed turbulent pipe flow can lead to considerable acoustic damping under certain conditions. Howe [4] proposed an empirical formula to model the effective viscous sublayer of turbulent flow, which depends on both the frequency of sound wave and the turbulence relaxation time. Based on this empirical model, and Prandtl's linear approximation, he suggested an eddy-viscosity model to relate the oscillation of Reynolds stress due to the presence of sound wave to the acoustic strain rate, and derived an analytical formula to predict the acoustic damping due to turbulent absorption, which is shown to compare well with experimental data [1, 4].

Recently Hamlington and Dahm [2, 3] studied the frequency response of initially isotropic homogeneous turbulence subjected to a periodic strain field and addressed a Reynolds stress closure for nonequilibrium effects in turbulent flows based on the exact dynamical equation for Reynolds shear stress anisotropy. The present authors adapted their model to the case of sound propagation in turbulent pipe flow and derived a new frequency dependent eddy-viscosity model accounting for the sound-turbulence interaction, which shows good agreement with the experimental data. This new model relates the interaction phenomenon straight to turbulence relaxation time scale, indicating that the effective viscous sublayer is not necessary to be introduced; in addition, Prandtl's linear approximation used in Howe's model could be replaced by other more precise equilibrium anisotropy closure.

MODELLING OF DAMPING OF ACOUSTIC WAVES

The formulation starts by decomposing the quantities of turbulent pipe flow containing sound wave into three parts (see, for example, [6, 7]):

$$F(\mathbf{x},t) = \overline{F}(\mathbf{x}) + \widetilde{F}(\mathbf{x},t) + F^{'}(\mathbf{x},t) \tag{1}$$

where $\overline{F}$ is the mean (time-averaged) contribution which is assumed to be time independent, $\widetilde{F}$ is the "statistical" contribution of sound wave, and $F^{'}$ corresponds to the turbulent motion. Applying this decomposition to the equations for conservation of mass, momentum, energy and ideal gas law, together with the treatment of first order approximation of acoustic quantities, one could obtain the governing equations for sound wave. Differing from the classical wave equations in pipe flow where the losses are caused solely by visco-thermal effects , the derived equations here includes an extra term describing the the oscillation of the background Reynolds stress due to the presence of sound wave, given by

$$\widetilde{r}_{ij} = \left\langle u_i^{'} u_j^{'} \right\rangle - \overline{u_i^{'} u_j^{'}} \tag{2}$$

where $\left\langle u_i^{'} u_j^{'} \right\rangle$ denotes the phase average of $u_i^{'} u_j^{'}$, and $\overline{u_i^{'} u_j^{'}}$ the time average. Since $\widetilde{r}_{ij}$ is unknown, we have a closure problem in the equations.

A simple model, called Quasi-laminar model, simply puts $\widetilde{r}_{ij} = 0$. This is equivalent to assuming that the turbulence interacts with sound waves via only mean velocity profile, not through its stresses. Results show that the Quasi-laminar model fails to predict the character of acoustic damping caused by turbulent absorption, which emphasizes the importance of $\widetilde{r}_{ij}$ in the procedure of sound-turbulence interaction. Another model, called Newtonian eddy model, relates $\widetilde{r}_{ij}$ to the acoustic strain rate $\widetilde{S}_{ij}$ through a scalar eddy viscosity ε , i.e., $\widetilde{r}_{ij} = -2\varepsilon\widetilde{S}_{ij}$. We employed a widely used expression for ε, which was originally suggested for pipe flow by Cess, to determine $\widetilde{r}_{ij}$ in our simulations but the results were only in qualitative accordance with the experimental data. The reason is that the Newtonian eddy model is an equilibrium closure which is unable to account for the nonequilibrium effect in turbulent flow due to the passage of sound wave. Following ref. [2, 3], we derived an modified Newtonian eddy model including the nonequilibrium effect as

$$\widetilde{r}_{ij} = -\frac{\varepsilon}{1 + i2\pi f \Lambda_m}\widetilde{S}_{ij} \tag{3}$$

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where Λ_m is the turbulence memory time scale, and f the frequency of sound wave.
Compared with Howe's model, this new model provides a more theoretical way to add turbulent absorption to wave equations.

RESULTS

The wave equations including the modified Newtonian eddy model eq. (3) are solved numerically. Assume low Mach number pipe flow which simplifies the problem by neglecting the compressibility of the mean flow. The frequency of sound wave under consideration is below the cut-on frequency of the first non-plane wave mode, so plane wave solutions could be obtained. No slip and iso-thermal boundary conditions on the wall of the pipe are adopted.
The comparisons of the simulated results and the experimental data [1], together with the results computed with the Quasi-laminar model and the unmodified Newtonian eddy model, are shown in figure 1. As can be seen, the modified Newtonian eddy model eq. (3) gives better agreement with the experimental data.

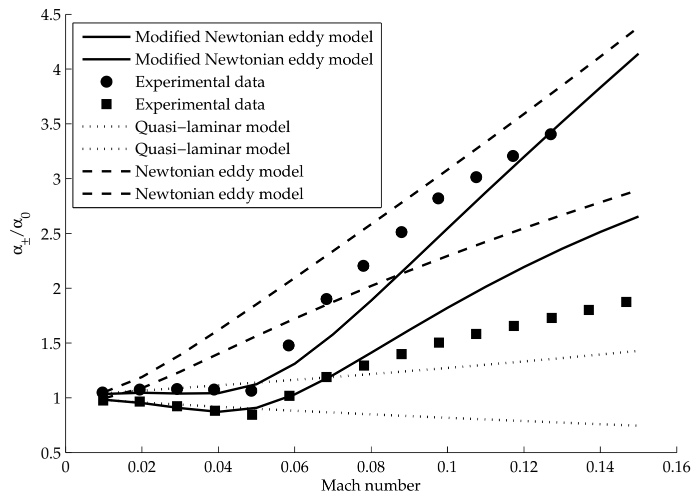

Figure 1. Relation between normalized acoustic damping $\alpha_\pm/\alpha_0$ and Mach number for Helmholtz number 0.032. The upper and lower curves correspond the upstream and downstream cases respectively.

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