ON THE GENERALIZED PHILLIPS' SPECTRA FOR WIND WAVES

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Summary A generalization of the kinetic equation by Newell & Zakharov [1] is used for explaining observed shapes of wind wave spectra. The approach allows to fix a critical uncertainty in modeling wind wave spectra using a trivial condition of equilibrium of nonlinear transfer and wave dissipation due to breaking of relatively short and steep waves. The proposed function of wave dissipation depends on wave steepness in power 6. The results of extensive simulation with the new dissipation function are presented in order to detail physics of the Phillips spectrum $E \sim \omega^{-5}$, its relevance to the proposed theory and to available experimental evidences.

INTRODUCTION

Wind waves usually observed in seas and lakes represent an extremely complicated natural phenomenon that is governed by a great number of physical mechanisms. Complete physical theories for majority of these mechanisms are not proposed so far that makes particular asymptotic cases extremely valuable. Phillips [2] considered the limiting case of saturation of a random wave field due to wave breaking and found his famous energy spectrum $E(\omega) = \alpha g^2 \omega^{-5}$ from dimensional consideration. Further studies showed that the Phillips constant α is not constant at all and the spectrum exponent -5 is not a universal one as well. The conceptual passage from the idea of saturation to the concept of an equilibrium of random wave field brought Phillips himself to the conclusion that the equilibrium can provide spectra exponents other than -5 [3]. The essential flaw of the Phillips theory of spectral equilibrium, in its own words [3], is a number of empirical coefficients responsible for balance of three principal constituents of wind-driven sea balance: nonlinear transfer, wave input by wind and wave dissipation due to variety of physical mechanisms like turbulence, mixing and, again, effect of the wind flow. In this paper we propose a physical model of wind wave spectra basing on extension of conventional statistical description within the kinetic (for wind waves – the Hasselmann) equation. This extension has been introduced recently in a general form by Newell & Zakharov [1] in their attempts to describe a balance of nonlinear transfer due to resonant wave-wave interactions and intermittent dissipation dealing with wave breaking. The key feature of the approach: the condition of the nonlinear-dissipative equilibrium fixes the dissipation rate that provide the stationary Phillips spectrum. Thus, in contrast to the Phillips idea of wind-wave balance the model does not allow any free parameter for the physical model of wave dissipation. The simple model allows for introducing a new dissipation function that can be expressed in terms of wave steepness and, then, simulated and compared with its counterparts used in wind-wave modeling.

THE MODIFIED KINETIC EQUATION FOR THE NONLINEAR-DISSIPATIVE PHILLIPS SPECTRUM

The authentic kinetic equation for deep water waves derived by Hasselmann [4] does not contain wave input and dissipation. These terms are introduced basing on additional models describing a variety of processes of wind-wave, wave-turbulence *etc.* coupling. Constructing our extension of the kinetic equation we introduce dissipation effects in terms of physical quantities of the authentic model. We treat the dissipation as divergence of spectral flux that depends on the spectral flux itself. In terms of frequency dependent energy spectrum $E(\omega)$ and energy spectrum flux $P(\omega)$ one gets [1]

$$\frac{\mathrm{d}E(\omega)}{\mathrm{d}t} = -\frac{\partial P(\omega)}{\partial \omega} - \Psi(P\omega^3/g^2)\frac{P(\omega)}{\omega} \tag{1}$$

with an arbitrary function Ψ of non-dimensional variable $P\omega^3/g^2$. The physical sense of the value can be acquired easily from dimensional consideration and homogeneity properties of the collision term S_{nl}. For gravity waves one has

$$P\omega^3/g^2 \sim E^3(\omega)\omega^{15}/g^6 = \mu_w^6 \tag{2}$$

i.e. the non-dimensional argument of Ψ is proportional to wave steepness in power 6. Thus, our physical model of wave dissipation reproduces the Phillips idea – dissipation is stimulated by wave steepening. While the authentic Phillips model of saturated spectrum [2] implies dissipation due to breaking events at infinitely short times, the model (1) operates with finite evolution time scales. Stationary solutions for (1) can be obtained easily, say, for power-like functions $\Psi(P\omega^3/g^2) = C(P\omega^3/g^2)^R$. One has two important results. First, the dissipation rate is fixed for any exponent $R \neq 0$

$$C\left(P\omega^3/g^2\right)^R = 3 \tag{3}$$

Secondly, the homogeneity relationships gives immediately the Phillips spectrum

$$E(\omega) = \alpha_{Ph} g^2 \omega^{-5} \tag{4}$$

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For $R = 0$ the fixed dissipation rate (3) and the Phillips spectrum exponent (-5) are guaranteed by homogeneity properties of the collision term $S_{nl} = -\partial P/\partial \omega$ [5]. This case is of key importance as far as it provides a family of self-similar solutions [see 6]. The Phillips parameter α_{Ph} in (4) should be determined from the boundary conditions for the modified kinetic equation (1). The conditions of continuity of spectrum $E(\omega)$ and spectral flux P can be imposed at a breakdown frequency ω_{diss} where waves start to break and the model (1) becomes valid. With homogeneity relationships (2) the dissipation term can be presented in the form of step-like function of wave steepness μ_w as follows

$$S_{diss} = C_0 \mu_w^4 \omega E(\omega) \Theta(\omega - \omega_{diss}). \tag{5}$$

Here where $\Theta(x - x_0)$ is the Heaviside function. The property of self-similarity at $R = 0$ appears to be of great value when trying to fit parameters of the modified and the authentic non-dissipative kinetic equations. The dissipation coefficient $C_{Phillips}$ appears to be independent on the breakdown frequency ω_{diss} and can be calculated explicitly for the case of isotropic spectra [5]. Its value $C_0 = C_{Phillips} \approx 2.19$ provides the Phillips spectra $E(\omega) \sim \omega^{-5}$. Thus, the dissipation function (5) can be regarded as a theoretically based one that explains observed features of wind-wave spectra. It should be noted, that dissipation function S_{diss} (5) for $C_0 \neq C_{Phillips}$ can generate any spectral exponents but (-4). The latter corresponds to the classic Kolmogorov-Zakharov solution when the collision term S_{nl} is plain zero.

SIMULATION OF WIND-WAVE GROWTH WITH THE NEW DISSIPATION TERM

Extensive numerical study of wind-wave growth has been carried out for conventional parameterizations of wind forcing and the new model of wave dissipation presented above. In addition to S_{diss} expressed by (5) different forms of S_{diss} with breakdown in wave steepness and without any breakdown have been tested. All the numerical experiments showed robustness of the proposed physical model and quite weak dependence of integral characteristics of wave growth (total wave energy, peak frequency) on particular choice of the breakdown parameters. These parameters can affect the domain of the Kolmogorov-Zakharov direct cascade solution $E(\omega) \sim \omega^{-4}$ only, that, typically, occurs in the range $1.5 - 4$ frequencies of spectral peak ω_p as an intermediate asymptotics. High-frequency Phillips' asymptotics $E(\omega) \sim \omega^{-5}$ is found to be determined by correct choice of the dissipation coefficient C_0 in (5). The theoretical estimate of the coefficient $C_0 = C_{Phillips} \approx 2.19$ for isotropic stationary solutions of (1) appears to be consistent with results of simulations of anisotropic and essentially non-stationary solutions of the kinetic equation.

CONCLUSIONS

In this work we proposed to combine effects of nonlinear interactions due to four-wave resonances and the effect of essentially nonlinear dissipation within the framework of an extension of the kinetic equation to broader range of wave scales. In terms of spectral fluxes wave energy leakage is assumed to be proportional to the flux itself. Reformulation of the model in terms of wave amplitudes (wave steepness) reveals a transparent physical analogy with ideas of Phillips and with conventional vision of wave dissipation through intermittent process of wave breaking. The proposed dissipation function is strongly nonlinear in wave amplitude (as power 6 of differential steepness μ_w). The features of self-similarity allows to fit solutions in "purely nonlinear" and "nonlinear-dissipative" frequency ranges in a unique way with no free parameters like breakdown frequency of the dissipative range ω_{diss} in (5). The corresponding dissipation coefficient associated with the Phillips spectrum $E(\omega) \sim \omega^{-5}$ is estimated theoretically and found to be consistent with the observed features of wave spectra at high frequencies and with results of extensive simulation of wave growth. Note, that the proposed model of wave dissipation is in dramatic contrast with conventional parameterizations of wave dissipation in models of wave forecasting that rely mostly on mean over spectrum steepness parameter like $\mu_p^2 = E\omega_p^4/g^2$ (E – total wave energy).

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