EQUILIBRIUM DYNAMICS AND STATISTICS OF GRAVITY-CAPILLARY WAVES

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Summary The theory of Fedorov & Melville (1998) is used to study the growth rate of wind-forced gravity-capillary waves and to relate statistical descriptions of the surface to the dynamics. The dependence of the growth rate of the waves on the wave age agrees well with the theory of Miles and laboratory and field data collated by Plant (1982), and the mean square slope of the surface of the longer waves is dependent on the air-sea momentum flux in a manner consistent with the classical field data of Cox & Munk (1954).

INTRODUCTION

Despite their relatively small size when compared to the larger wind waves and swell, which are in the gravity-wave range, the occurrence of short, O(10) cm or less, gravity-capillary waves at the ocean surface is important for both dynamics and remote sensing. Dynamically the shorter waves may be of larger slope and therefore more inclined to break, leading to dissipation of the surface-wave energy. The generation of shorter parasitic capillary waves can also lead to significant viscous dissipation of energy which, in the absence of breaking, is usually insignificant for the longer waves. Parasitic capillary waves are generated by a direct resonance between short gravity waves and even shorter capillary waves having the same phase speed. These parasitic capillary waves are generated near the region of large curvature at the crest and propagate along the forward face of the longer gravity wave. Figure 1, from Fedorov et al. (1998) shows an example of a numerical calculation of the surface and surface slope for a gravity-capillary wave of 7 cm wavelength with parasitic capillaries on the forward face. In nature, the large fluctuations in the slope of the parasitic capillaries can lead to large densities of specular scatterers of sunlight from the ocean surface. The specular scattering was exploited by Cox & Munk (1954) in their classical paper that showed the mean square slope of the ocean surface, as measured by "sunglint", to be proportional to the wind speed: $\sigma_x^2 \propto \sigma_y^2 \propto U$. This result, which has recently been confirmed by global space-based measurements (Breon & Henriot, 2006), has been one of the foundations of remote sensing of the ocean surface from space, especially with regard to using the scattering of microwave radar ,"scatterometry", to infer ocean surface winds. In effect, it relates the geometry of the surface to the dynamics of air-sea interaction, especially to the air-sea momentum flux $\tau = \rho_a u_*^2 = \rho_a C_D U_{10}^2$, where ρ_a is the density of the air, u_* is the friction velocity in the air, C_D is the drag coefficient, and U_{10} is the wind-speed at 10 m above the mean sea surface.

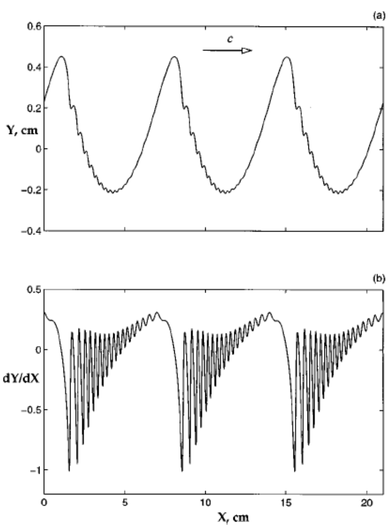

Figure 1: Numerical calculation of the profile of the surface (a) and its slope (b) for a wavelength $\lambda = 7$ cm. Note the different vertical and horizontal scales in (a). *(From Fedorov et al. (1998))*

Fedorov & Melville (1998) undertook a theoretical and numerical study of gravity-capillary waves with surface pressure forcing and viscous dissipation. Based on an outer irrotational flow in the water with a viscous boundary layer near the surface, the theory permitted the efficient calculation of steady gravity-capillary waves with parasitic capillary ripples. To balance the dissipation and achieve steady solutions, wind forcing was applied by adding a simple sinusoidal surface pressure distribution according to the phase of the longer wave with no modulation by the shorter capillary waves. It departed from previous studies in permitting fully nonlinear capillary waves that led to multi-valued solutions. While no wind-wave generation models were used to determine the pressure forcing, at least two classes of solutions were found: the first (Class 1) with the maximum pressure near the trough, consistent with Miles'(1957, 1959) mechanism of wind-wave generation; the second (Class 2) was near the crest, perhaps consistent with instabilities of the subsurface shear current. Steady solutions were also bounded by the amplitude of the pressure forcing. Measurements of mechanically generated gravity-capillary waves in a channel showed good agreement with the theory (Fedorov et al., 1998).

In this paper we explore whether the theory and solutions of Fedorov & Melville (1998) can be applied to interpret the dynamics and statistical descriptions of wind-generated gravity-capillary waves in the laboratory and field.

RESULTS

Solutions were computed for wavelengths in the range $0.03 \le \lambda \le 0.12$ m, with different wave slopes ak, where a is the wave amplitude, $k = 2\pi / \lambda$ the wavenumber, and for different mean momentum fluxes $\tau = \rho_a u_*^2$, due to the pressure forcing.

Since the solutions are steady in a frame of reference moving with the waves, the viscous dissipation of energy is balanced by the energy flux from the air due to the surface pressure distribution. Thus in the absence of viscous dissipation this energy flux would lead to a growth rate of the waves. Figure 2(L) shows that over the range of parameters considered, wavelength, pressure forcing and wave slope, this computed dimensionless growth rate of the waves β may be in agreement with laboratory and field data (Plant 1982) when plotted as a function of the reciprocal wave age u_* / c (c is the phase speed of the wave). The dimensionless growth rates also agree with Miles' theory as computed by Janssen (2004). It should be mentioned that Miles' theory does not account for parasitic capillaries. The agreement with the theory and observations suggests that spectra of short gravity waves in the laboratory and field may have the wind input balanced by the viscous dissipation associated with the parasitic capillaries rather than by breaking.

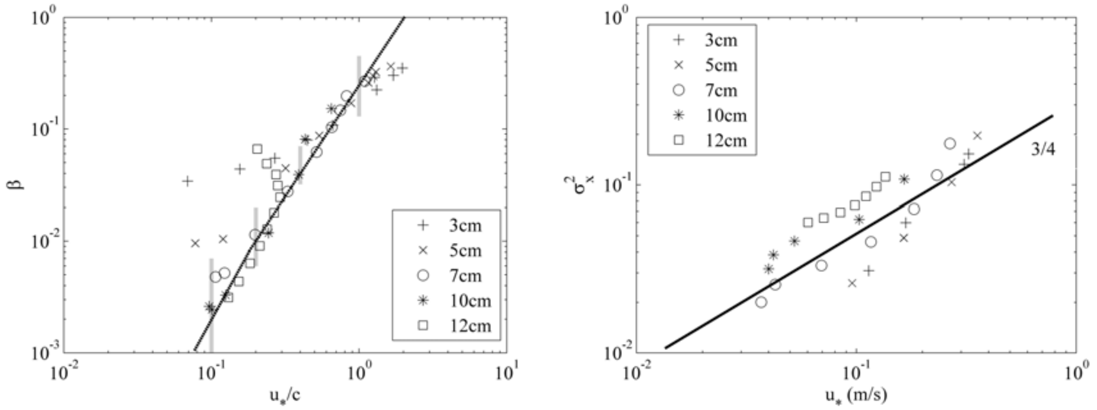

Figure 2: Left: Dimensionless wave growth rate β versus the reciprocal wave age compared with Miles' theory (solid line) and laboratory and field data collated by Plant (1982) within the gray bars. Right: Mean square slope of gravity-capillary waves versus u_*. The solid line has a slope of 3/4.

For gravity-capillary wave solutions that showed agreement with the wave growth data and Miles' theory, we examined the relationship between the mean square slope of the surface profile, σ_x^2, and the friction velocity, u_*. Figure 2(R) shows that for the longer waves, $\sigma_x^2 \propto u_*^{3/4}$, which for a drag coefficient, $C_D \propto U_{10}$, consistent with field data, would give $\sigma_x^2 \propto U_{10}^{9/8}$, very close to the result of Cox & Munk (1954). Directly applying solutions of the form described here to the field requires the assumption that patches of wind-driven gravity-capillary waves are of compact support at the ocean surface. The implication of these results for the fine-scale structure of the ocean surface will be discussed.

CONCLUSIONS

The theory of steady gravity-capillary waves by Fedorov & Melville (1998) is consistent with measurements and models of the dynamics and slope statistics of wind-generated waves in the laboratory and the field. This suggests that the wind input to the equilibrium ocean wave spectrum at wavelengths of O(10) cm may be balanced by the strong viscous dissipation due to parasitic capillary waves. This is expected to have important implications for the spectral and statistical description of the sea surface.

References

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