NONLINEAR LONG WAVES OVER A MUDDY BEACH

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Summary We describe an asymptotic theory on the interaction between water waves and a thin layer of fluid mud on a sloping beach. The depth of water is shallow and the waves are long. Weakly nonlinear effects in water are considered. The mud layer is modeled as a viscoelastic fluid. Using constitutive coefficients of mud samples from two field sites, we examine the interaction of nonlinear waves and the mud motion. It is shown that the waves experiences slow attenuation at the first order and harmonic generation at the second order, as the result of the first-order motion in mud. Instead of the Eulerian streaming well known in Newtonion visous fluids, a steady displacement is found in mud at the second order.

THE PROBLEM

There are many sea coasts in the world where the seabed is covered with fluid-mud which is a mixture of water, highly cohesive clay particles and sand. Dissipation in the mixture is considerably higher than in the pure sea water and causes strong damping of incident waves. In turn the waves can induce changes of the muddy seabed and alter the coastal morphology in the long run. For oscillatory flows available experiments have revealed that concentrated fluid-mud is quite viscoelastic instead of Bingham-plastic common in steady river flows. In this article we report a srudy of weakly nonlinear long waves over a shallow beach of gentle slope covered by a layer of viscoelastic fluid-mud. The constitutive coefficients are from mud samples taken from two field sites. An asymptotic theory is developed for the evolution and attenuation of time-periodic waves and the associated motion in mud. Numerical results will be presented to show the effects of different rheology as well mud depth and slope length, etc. Finally we examine the second-order time-averaged effect in the mud layer and derive the shoreward displacement due to waves which counters the seaward displacement due to gravity. This result distinguishes fluid-mud from a Newtonian viscous fluid where the time-averaged effects is instead a steady Eulerian current.

THE APPROACH

We assume the still water depth $h(x)$ to be a slowly varying function of x only, and the fluid-mud is immiscible and of constant depth d at rest. Periodic waves propagate normally from the open sea toward the shore. Let A, k and h_0 be the characteristic wave amplitude, wavenumber, and water depth respectively. The following ratios $\epsilon = A/h_0$, $\delta = d/h_0$, and $\kappa = kh_0$ which characterize respectively the wave amplitude, mud-depth and wavenumber are all assumed to be small and comparable in order of magnitude.

Following [1] the generalized viscoelastic model relating rate of stress to the rate-of-strain is adopted. For sinusoidal motion the effective viscosity $\mu(\omega)$ is a complex function of frequency. The constitutive coefficients are found by fitting data from dynamic tests of mud samples from Hangzhou Bay, China and Gulf of Mexico, for the frequency range of $0.1 < \omega < 10$ rad/s. It is found that the Hangzhou Bay mud is closer to being Newtonian, while the the mud from Gulf of Mexico is quite elastic.

Introducing fast and slow coordinates $x, X = \kappa x$ for describing wave fluctuations and spatial attenuation respectively, we expand, in dimensionless form, the vertical displacement of the free surface ζ and water mud interface η

$$\zeta = \frac{1}{2}\sum_{m=-\infty}^{\infty} A_m(X)e^{im\xi}, \quad \eta = \frac{1}{2}\sum_{m=-\infty}^{\infty} B_m(X)e^{im\xi}, \text{ where } \xi = \int^X dX/\sqrt{\kappa h(X)} - t$$

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The moving coordinate ξ follows the wave phase. Accounting for the second-order effects of wave motion, the spatial evolution of the free surface harmonics is found to be governed by the following nonlinear equations

$$\sqrt{h}\frac{dA_m}{dX}+\frac{h_{\mathfrak{X}}}{4\sqrt{h}}A_m+\frac{\epsilon}{\kappa}\frac{3i}{8h}m\left[\sum_{l=1}^{\infty}2A_l^*A_{m+l}+\sum_{l=1}^{[m/2]}\alpha_l A_l A_{m-l}\right] + \frac{im}{2}\frac{\delta}{\kappa}\frac{\gamma A_m}{h}\left[1-\frac{\tanh\sigma_m}{\sigma_m}\right]=0, \qquad m=1,2,3,... \tag{0.1}$$

After numerical solution for given initial amplitudes $A_m(0)$ at $X=0$, the interface amplitudes are then computed from

$$B_m(X)=\frac{\gamma A_m}{h}\left(1-\frac{\tanh\sigma_m}{\sigma_m}\right) \quad \text{where} \quad \sigma_m=\sqrt{-im\frac{\rho_M\omega d^2}{\mu_m'}} \tag{0.2}$$

By analyzing the second-order mean motion in mud, the drift displacement in the horizontal direction is also obtained. The corresponding vertical displacement implies change of mud layer thickness.

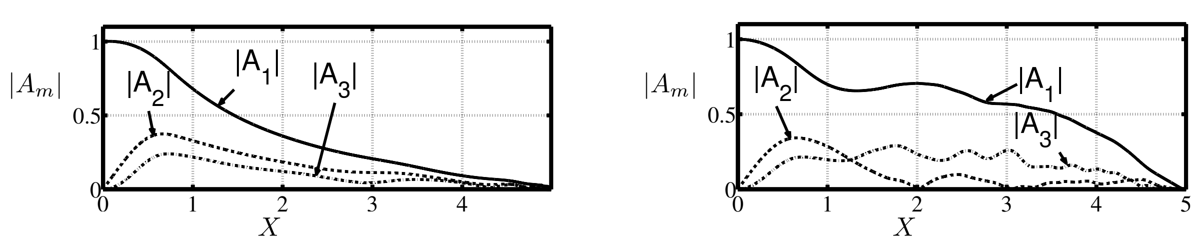
SAMPLE PREDICTIONS AND CONCLUSIONS

We have conducted extensive computations for the simple bathymetry where a plane beach is connected to an open sea of constant depth h_0. As an example Figure 1 gives the harmonic amplitudes $A_m(X)$ of the free surface for the following inputs : sea depth $h_0=3$ m, beach slope of $S=s\kappa=0.02766$, mud depth $d=0.3$ m. wave frequency $\omega=0.25$ rad/s (wave period $T=15.7$s), incident wave amplitude $A=0.45$m. The mud from Gulf of Mexico mud has the greater viscosity and regidity; the dominant harmonic $|A_1|$ is understandably smaller over most of the slope. Spatial undulations due to nonlinear energy exchanges among harmonics are greatly suppressed. In contrast, exchanges between harmonics due to nonlinearity are much more prominent in the less viscous and more Newtonian Hangzhou Bay mud. It is confirmed analytically that over a sloping beach attenuation by mud can be so effective that waves are damped out before reaching the shore. All the interface harmonics B_m (not shown here) are much smaller, implying weak motion in the mud layer, which must be caused by the much larger elasticity (or rigidity).

In a Newtonian viscous fluid, it is well known that there is a steady drift (Eulerian streaming) in the oscillatory boundary layer near the solid wall due to the nonlinear effect of convective inertia. Such an effect is of interest to sediment transport and is responsible for the formation of sand bars and affect the morphology of a sandy coast. At the second order nonlinearity in the visco-elastic mud gives rise to a time-averaged drift displacement similar to the drift current in the oscillatory boundary layer of a viscous fluid. The displacement induces a slight thinning of the mud layer offshore but slight thickening close to the shore.

References

[1] Mei, C. C., Krotov, M., Huang, Z. and A. Huhe, 2010, Short and long waves over a muddy seabed. *J. Fluid Mech* 643, 33-58.



(a) Gulf of Mexico mud

(b) Hangzhou Bay mud

Figure 1. Evolution of the first 3 harmonics of the free surface over different muddy seabeds. For $\epsilon=0.15$ and $s=1/5, \delta=0.1$