ON THE PAIRING INSTABILITY IN HELICAL VORTICES

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Summary The pairing instability in helical vortices is analysed theoretically and experimentally. In the theoretical study, the helical vortices are modelled by an infinite array of vortex rings. The spatio-temporal development of the pairing instability is analysed by linearized DNS as function of the geometric parameters of the vortices and the Reynolds number. In the experimental study, the helical vortices are created with a rotor in a water channel. Visualisations and PIV measurements of the pairing instability are performed. Applications to configurations relevant to wind turbines and helicopters are also discussed.

INTRODUCTION

The stability properties of helical vortices are of interest for applications such as wind turbine and helicopter wakes. In this work, we consider the particular instability that leads to vortex pairing, and we propose that global transitions observed in the wake of rotors, e.g. the transition to a Vortex Ring State of the wake of a helicopter in steep descent (Leishman *et al.* [1]), may possibly be associated with the change of nature from convective to absolute of this instability.
Numerical simulations have shown that several instabilities can occur in helical wakes. Both the three-dimensional elliptic instability and the two-dimensional pairing instability were found to play an important role in helicopter as well as wind turbine wakes (Walther *et al.* [2], Ivanell *et al.*[3]). However, the theoretical results known about the stability properties of a helical vortex wake were established for only few model cases: a helical filament (Widnall [4]) or in the limit of large wavenumbers (Okulov [5], Okulov and Sørensen [6]). For the pairing instability, which requires to take into account the geometry of the vortex core, a systematic analysis of the absolute/convective nature of the instability was performed numerically in the case of an array of straight vortices (Brancher and Chomaz [7]) but has still to be done for a helical vortex wake.

NUMERICAL STUDY

In the first part of the present study [8], helical vortex systems with small pitch h (compared to the helix radius R) were modelled by an infinite array of vortex rings. The choice of this axisymmetric configuration eliminates potential three-dimensional instabilities of the system and allows focussing on the pairing instability. For small helix pitches, this provides a good approximation of the azimuthal vorticity field, as shown in Figure 1.

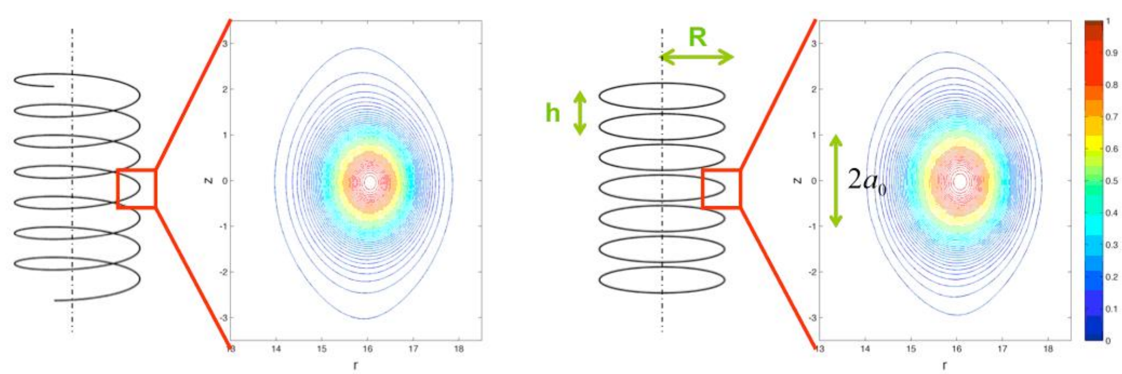

Figure 1. Sketches and azimuthal vorticity contours of a helical vortex (left) and of a vortex rings array (right). The initial condition in both cases is a Gaussian vortex of core radius a_0 and with no axial velocity. Simulations were run for $\frac{a_0}{R} = \frac{1}{16}$ and $\frac{a_0}{h} = \frac{1}{8}$ at $Re = \frac{\Gamma}{2\ \pi\ \nu} = 500$. The helical vortex is computed by forcing the flow to have a helical symmetry (see Delbende [9]).

Numerical simulations using the linearized Navier-Stokes equations were used to investigate the spatio-temporal growth of this instability, following a localized initial perturbation, as function of the geometric parameters of the wake (a_0/h and a_0/R, where a_0 is the vortex core size) and the Reynolds number Re based on the vortex circulation.
When adapted to the cases of wind turbine and helicopter wake, these results allow, respectively, the determination of a characteristic wake decay distance, and the identification of configurations in which the pairing instability may be able to trigger a global transition.

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EXPERIMENTAL STUDY

In addition to the theoretical study, experiments are carried out with a rotor in a water channel (see figure 2). Results from dye visualizations and measurements using Particle Image Velocimetry will be presented, giving a precise qualitative and quantitative description of the spatio-temporal development of the pairing instability in a realistic helical vortex wake under well-controlled conditions. Comparison with the theoretical predictions will also be made.

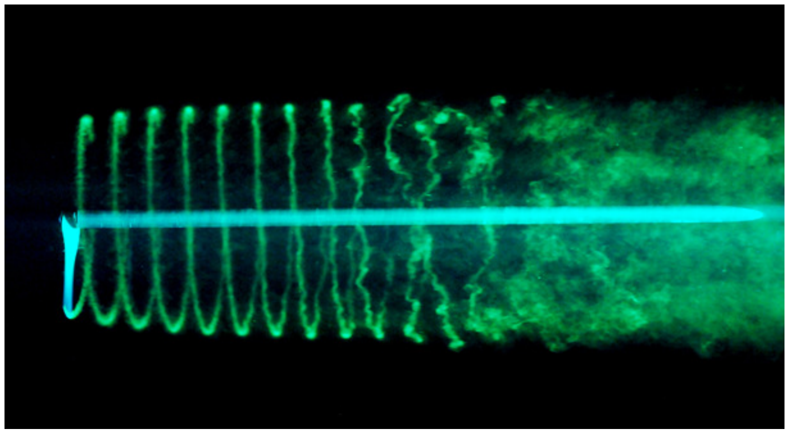

Figure 2. Dye visualization of a single-blade rotor wake in a water channel. The rotor of radius 8 cm is designed to have constant circulation (Joukowsky rotor) for a tip speed ratio of 13.4, which is the case shown here. Several instabilities at different scales can be identified, including local pairing of successive vortex loops.

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References

[1] Leishman J. G., Bhagwat, M. J. and Ananthan, S. The vortex ring state as a spatially and temporally developing wake instability. *J. Amer. Helicopter Soc.*, 49:160–175 , 2004.
[2] Walther J.H. *et al.* A numerical study of the stability of helical vortices using vortex methods. *J. Phys. Conf Series*, 75:012034, 2007.
[3] Ivanell, S., Mikkelsen, R., S£rensen, J. N. and Henningson, D. Stability analysis of the tip vortices of a wind turbine. *Wind Energy* 13:705-715, 2010.
[4] Widnall S.E. The stability of a helical vortex filament. *J. Fluid Mech.*, 54:641–663, 1972.
[5] Okulov V.L. On the stability of multiple helical vortices. *J. Fluid Mech.*, 521:319–342, 2004.
[6] Okulov V.L. and Sørensen J.N. Stability of helical tip vortices in a rotor far wake. *J. Fluid Mech.*, 576:1–25, 2007.
[7] Brancher P. and Chomaz J.M. Absolute and convective secondary instabilities in spatially periodic shear flows. *Phys. Review Letters*, 78:658–661, 1997.
[8] Bolnot, H., Le Dizès, S. and Leweke, T. Spatio-temporal development of instabilities in helical vortices. *AIAA Paper* 2011-3927, 2011.
[9] Delbende I. DNS of vortices with helical symmetry. *Proceedings of the* 8^{th} *Euromech Fluid Mechanics Conference*, 2010.