CONSTRUCTION AND EVOLUTION OF VORTEX-SURFACE FIELDS

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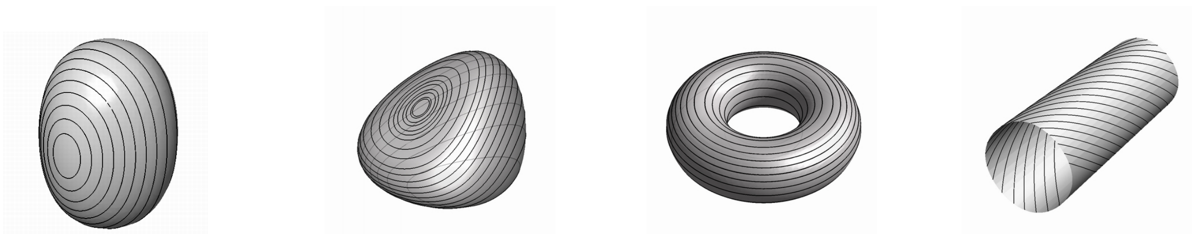
Summary We develop a Lagrangian-like formulation, together with numerical methods, for the construction and evolution of vortex-surface fields in viscous Taylor–Green and Kida–Pelz flows. Numerical simulations show the collapse of vortex surfaces followed by vortex reconnection and the formation and roll-up of vortex tubes which are subject to vortex stretching and twisting. These results enable the visualization, in terms of the changing topology of vortex surfaces, of transition from a laminar to a turbulent-like flow.

Introduction

Vortex tubes and sheets have long been utilized as the conceptual elements of the vortex-dynamic paradigm in fluid mechanics. In particular, intermittency in high-Reynolds-number turbulence has been hypothesized to be related to the organized vortical structures with candidate tube-like or sheet-like geometries [1]. In a general sense, an open or closed "vortex surface" (or "vorticity surface") can be defined as a smooth surface or manifold embedded within a three-dimensional velocity field. The local vorticity $\boldsymbol{\omega}$ is tangent at every point on the vortex surface. Although coherent bundles of vortex lines strong vorticity have been observed in turbulence, the existence of globally smooth vortex surfaces in general three-dimensional flows remains an open problem. For strictly inviscid barotropic flow with conservative body forces, the Helmholtz vorticity theorem shows that Lagrangian material surfaces which are vortex surfaces at time $t = 0$, remain so for $t > 0$. For viscous flow, on the other hand, the breakdown of the frozen motion of vortex lines owing to violation of the Helmholtz theorem underscores a fundamental problem of identifying vortex surfaces at different times [2]. This could be interpreted as the root cause of numerical obstacles in the implementation of Lagrangian formulations of Navier–Stokes dynamics. In this study, we develop a Lagrangian-like formulation to reveal vortex dynamics in the sense of the geometry and topology of vortex surfaces in incompressible inviscid/viscous flows.

Construction of initial vortex surface fields

Within the Clebsch representation, we define a vortex-surface field (VSF) ϕ_v as a smooth scalar field whose iso-surfaces are vortex surfaces [3]. A systematic methodology for the construction of VSFs for both Taylor–Green (TG) [4] and Kida–Pelz (KP) [5] initial velocity fields is developed. The spatial constraint $\lambda_\omega \equiv \boldsymbol{\omega} \cdot \nabla\phi_v = 0$ on a three-dimensional VSF can be converted to a system of linear homogenous equations with Fourier expansions. In particular, although it appears that we cannot find an exact and unique ϕ_v for the initial KP velocity field with a finite set of Fourier modes, a least-squares solution through an optimization problem with proposed basis functions satisfying all KP symmetries can be constructed. Examples of vortex surfaces as iso-surfaces of ϕ_v of several simple flows at $t = 0$ are shown in Fig. 1.



(a) TG flow (b) KP flow (c) vortex ring (d) ABC flow (integrable)

Figure 1. Typical vortex surfaces and vortex lines in simple flows at the initial time.

Evolution of vortex surface fields

With an initial VSF, ϕ_v is a Lagrangian scalar field [6] in inviscid flow. For viscous flow, we expand the space of independent co-ordinates of a set of equations that formally describe the temporal evolution of a VSF, to an extended domain containing a pseudo-time co-ordinate [7]. This appears to restore uniqueness to the indeterminate initial boundary-value problem and can be interpreted as a correction step on the original VSF governing equations [6]. In the absence of a physical diffusion term, solutions of the extended equation set for VSFs are expected to develop exponentially small scales in the long pseudo-time limit. In the current numerical implementation, this is treated through regularization provided by the implicit numerical dissipation inherent in the numerical WENO scheme used. Using this numerical method, satisfactory convergence in terms of the pseudo time up to a sufficiently small error threshold for λ_ω is obtained.

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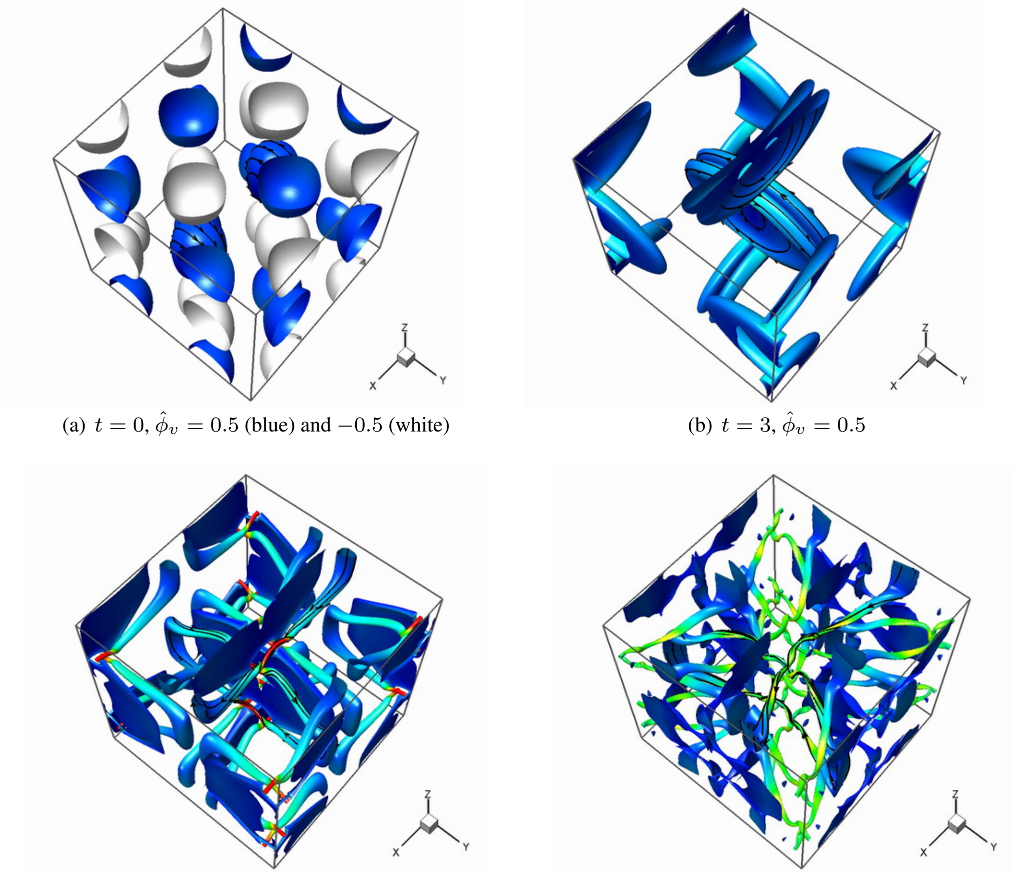
The present numerical scheme has been applied in TG and KP flows to evolve the VSF governing equations for real-time periods, and smoothed vortex surfaces are extracted as iso-surfaces of the continuous VSFs. As shown in Fig. 2, the continuous evolutionary geometry of vortex-surface evolution from smooth, laminar-like behavior flows through a "transition" to a turbulent-like state is revealed in a long enough simulation duration. The signature of this transition is the onset of topological and geometrical changes in the vortex-surface structure near $t = 5$. Pictorially this corresponds to some typical scenarios of vortex dynamics in turbulence, e.g. the collapse of vortex surfaces and vortex reconnection (see Fig. 2(b)), rolling-up of vortex tubes, vorticity intensification between anti-parallel vortex tubes (see Fig. 2(c)), and vortex stretching and twisting (see Fig. 2(d)).



(a) $t = 0$, $\hat{\phi}_v = 0.5$ (blue) and -0.5 (white)

(b) $t = 3$, $\hat{\phi}_v = 0.5$

(c) $t = 5$, $\hat{\phi}_v = 0.5$

(d) $t = 7$, $\hat{\phi}_v = 0.45$

Figure 2. Evolution of vortex surfaces in the TG flow at $Re = 400$. Some vortex lines are integrated on the iso-surfaces at $\hat{\phi}_v$. Color on the surfaces is rendered by $0 \leq |\boldsymbol{\omega}| \leq 12$ from blue to red, where $\hat{\phi}_v \equiv \phi_v/|\phi_v|_{max}$. The iso-surfaces of ϕ_v at negative contour levels are not shown for clarity except in Fig. 2(a).

Conclusions

In the present study, we develop a Lagrangian formulation of vortex dynamics, which enables, in a novel way, the observation of the timewise evolution of smoothed VSFs in some simple viscous flows. It is noted that globally smoothed VSFs may not exist in general flows when the vorticity field is not integrable, in which case a surrogate VSF could perhaps be obtained.

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