SYNCHRONIZATION OF TWO SPRING-LOADED CYLINDERS IN A UNIFORM FLOW

Makoto Iima* & Masaki Nakasuji**
* *Department of Mathematical and Life Sciences, Graduate School of Science, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan*
** *Department of Mathematics, Graduate School of Science, Hokkaido University, Sapporo, 060-8628, Japan*

Summary We have studied the synchronization between two spring-loaded cylinders aligned transversely to a uniform flow as a fundamental problem of hydrodynamic interaction between objects moving through internal mechanics. Depending on the distance between two cylinders and the characteristic frequency of the spring-loaded cylinder, both in-phase and out-of-phase type synchronization have been observed, although the phase between two cylinders has a deviation from 0 and π, respectively. We have also applied a phase reduction technique to describe such synchronized behaviour by a phase equation.

INTRODUCTION

Animals such as butterflies and fish drive their locomotive organs (wings and fins) periodically to drive fluid. Generated hydrodynamic force is used to move or to control their motion. In the moderate Reynolds numbers, generated vortices show spatio-temporal patterns, which determine the characteristics of the hydrodynamic force. Even for objects with simple geometry like cylinder, the oscillating object in a uniform fluid shows a variety of vortex patterns.

Because of nonlinearity, vortex patterns by two or more oscillating objects becomes much complex, that is, the generated force has also complicated characteristics. Thus, the hydrodynamic interaction between locomotive organs (e.g., wings of single butterfly and fins in fish schooling) affects their ability of flight or swimming. A control of driving organs, either active or passive, can lead to a coherent vortex pattern, which is important to stabilize their collective motion. On the other hand, another way of control may lead to a non-coherent vortex pattern, which might be useful for quick or unpredictable direction change for animals. To consider this problem, the coupling effects between object-driving dynamics and fluid dynamics should be studied.

In this paper, we study two spring-loaded cylinders aligned transversely to a uniform flow as a fundamental problem of hydrodynamic interaction between objects moving through internal mechanics. Depending on the distance between two cylinders and the characteristic frequency of the spring-loaded cylinder, both in-phase and out-of-phase type synchronization have been observed. We have also applied a phase reduction technique to describe such synchronized behaviour by a phase equation.

PROBLEM FORUMULATION

We consider two spring-loaded cylinder with diameter d (C_1, C_2) aligned transversely to a uniform flow $\boldsymbol{U}=(U,0)$ (Fig.1(a)). Each cylinder C_i (i=1,2) oscillates in the y-direction alone around $(0,Y_{i0})$ with characteristic frequency f_i if there is no fluid. Here we do not assume any dumping and external force except for fluid effect for simplicity. The governing equation for fluid is the incompressible Navier-Stokes equations. The equation of motion for the cylinder is:

$$m\frac{d^2Y_i(t)}{dt^2} = -m(2\pi f_i)^2(Y_i(t)-Y_{i0}) + F_h(t),$$

where m is the mass and F_h(t) is the hydrodynamic force acting on the cylinder.

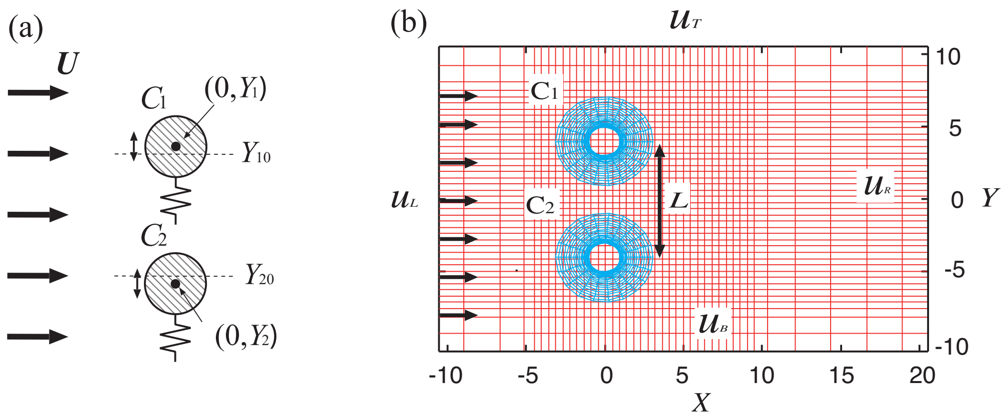

Figure 1

For numerical simulation, we used a finite volume method with a fortified solution algorithm. The whole computational domain was discretized by a non-uniform Cartesian grid, and the domains near cylinders are discretized by a cylindrical grid. The boundary conditions are $\boldsymbol{u}=(0,dY_i/dt)$ on C_i; $\boldsymbol{u}_L = \boldsymbol{u}_T = \boldsymbol{u}_B = (0,U)$, $\partial \boldsymbol{u}_R/\partial n = 0$ (Fig.1(b)). The parameters are $m=7$, $d=2$, $U=1$ and $\nu=0.005$ (Re=400) to obtain Kármán-vortex like symmetric pattern for single cylinder (S-type pattern). We controlled L, distance between two cylinders, and f_i.

RESULTS

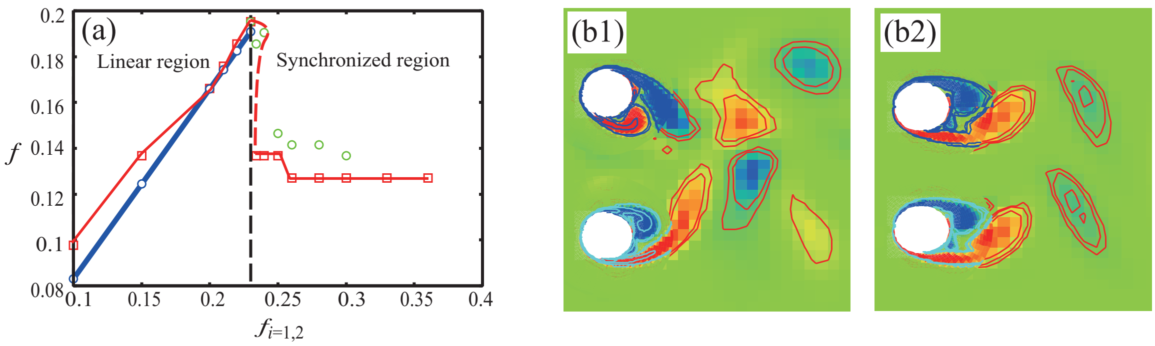

Figure 2

Fig. 2(a) shows the observed frequency of cylinder oscillation f as a function of characteristic frequency f_i for single cylinder case. When $f_i < f_K$ (where f_K =0.234 is the frequency of the wake for single fixed cylinder in which the Kármán vortex street is observed), f is roughly linear to f_i, and the gradient can be explained by the added mass effect (blue line). When $f_i > f_K$, f is almost constant and close to $f_K/2$ (=0.117), where entrainment to the Kármán vortex street is observed. In Fig. 2, both out-of-phase (f_1=0.15; (b1)) and in-phase (f_1= 0.3; (b2)) synchronizations of oscillating cylinders are shown ($\lambda = L/d$ = 2.5 and $f_2/f_1=\alpha=1.05$). In both cases, they are synchronized, but the actual phase differences were 0.8π (b1) and 0.1π (b2), respectively.

The synchronized regions for different distances are investigated (f_1=0.15; Fig. 3(a)). The synchronized region becomes narrower as $1/\lambda$ becomes smaller, which is suggested by the synchronization theory. If the hydrodynamic interaction between cylinders is weak, the coupling dynamics of the cylinder should be described by a phase equation:

$$\frac{d\phi_1}{dt} = 2\pi f_1 + \Gamma(\phi_2 - \phi_1), \frac{d\phi_2}{dt} = 2\pi f_2 + \Gamma(\phi_1 - \phi_2),$$

where Γ is the coupling function. In Fig. 3(b), the shape of Γ is shown for the case α=1.055 and $1/\lambda$ = 0.4 where the synchronization is slightly broken, which was obtained by the algorithm in Ref. [1].

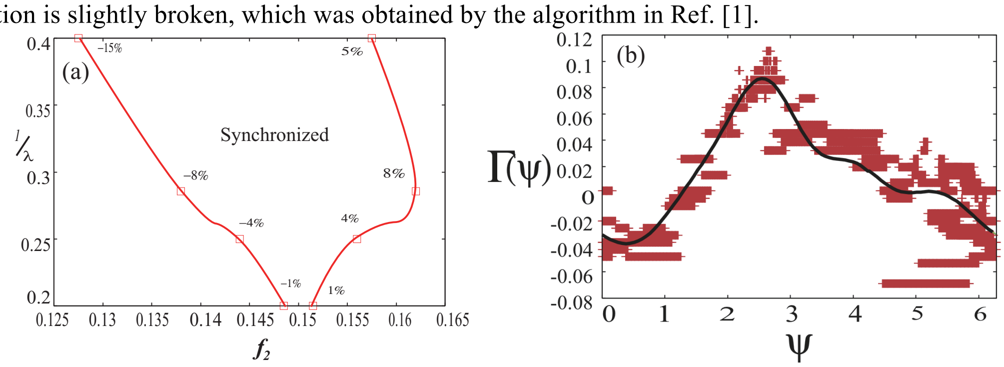

Figure 3

CONCLUSIONS

We have studied synchronization of spring-loaded cylinders in a uniform flow. Even if the cylinder is not entrained in the vortex street generated by cylinder, synchronization between oscillating cylinders is observed. Such dynamics can be represented by a phase equation, and the shape of the coupling function was obtained by the direct numerical simulation data.

References

[1] Miyazaki J. and Kinoshita S.: Determination of a Coupling Function in Multicoupled Oscillators. *Phys.Rev. Lett.* **96**:194101, 2006.