APPLICATION OF MOSAIC-SKELETON APPROXIMATIONS METHOD TO ACCELERATE CALCULATIONS IN THE PROBLEMS OF MODELING FLOWS PAST BODIES WITH DISCRETE VORTICES METHOD

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Summary This paper is related to the problem of calculations acceleration in the vortex method for solving 3D unsteady hydrodynamic problems by applying mosaic-skeleton approximations method to find velocity field. Author proposed modified numerical scheme of vortex segments which allows effective application of mosaic-skeleton approximations of matrices which appear in the problem of the evolution of 3D vorticity field in unbounded domain and in the problem of modeling separated flow past bodies.

INTRODUCTION

Vortex methods based on the Lagrangian approach are widely used in modern fluid dynamic simulations. These methods determine the trajectories of fluid particles, and the flow characteristics of fluid particles are regarded as functions of their initial positions and time. A specific feature of these methods is that they use an integral representation of the velocity field in terms of vorticity on a moving grid. Moreover, every integration step in time includes a transformation of the unknowns that can be reduced to the multiplication of a matrix by a vector.

At present, the most interesting possibility of accelerating discrete vortex methods is provided by fast matrix multiplication, for example, by mosaic-skeleton approximations [1].

In this paper, a modified numerical scheme for the vortex segment method [2,3,4] is proposed in which mosaic-skeleton matrix approximations are effectively applied to the simulation of vorticity transport in an unbounded 3D domain and to separated flow problems. Several aerodynamic problems are used to numerically study the acceleration of the computations, the approximation of solutions, and the choice of a priori parameters in mosaic-skeleton approximations.

MODELING FLOW OF IDEAL FLUID PAST BODY

The 3D problem of unsteady flow of ideal incompressible fluid with given velocity on infinity past body (group of bodies) is considered. At body surfaces the non-penetration or known-law penetration condition is set. It is assumed that vortex wake separate from predefined separation lines on bodies. Vortex wake shifting submits to the law of moving vortex lines together with fluid particles and the law of constant circulation over moving together with fluid contour.

To create the numerical scheme for the described problem authors use the modified version of discrete vortices method [3,4]. Vortex wake is divided in two parts “near” and “far”. Near wake consists of vortex frames whish separate to the flow in discrete moments of time and their circulations do not change in time. Later on the frames generating near wake can be regrouped to the set of vortex segments with connected structure. Far wake consists of isolated vortex segments which can be disconnected from each other and their length do not change in time.

Velocity in any point of flow can be calculated by formula $\mathbf{W} = \mathbf{W}_T + \mathbf{W}_P + \mathbf{W}_V + \mathbf{V}_\infty$, where $\mathbf{W}_T$ - velocity induced by frames on body, $\mathbf{W}_P$ - velocity induced by connected vortex segments from near wake, $\mathbf{W}_V$ - velocity induced by isolated segments, $\mathbf{V}_\infty$ - velocity on infinity. To calculate velocity induced by vortex segments in the point $\mathbf{x}$ the following formula is used

$$\mathbf{W}_\varepsilon(\mathbf{x}) = \frac{1}{4\pi}\sum_{j=1}^{n}\frac{\Gamma_j \mathbf{l}_j \times (\mathbf{x} - \mathbf{y}_j)}{\left|\mathbf{x} - \mathbf{y}_j\right|^3} \bullet \theta_\varepsilon(\left|\mathbf{x} - \mathbf{y}_j\right|), \tag{1}$$

Where $\mathbf{l}_j^0$ - vortex segment with known beginning and the end, Γ_j - intensity of vortex segment concentrated in its center, $j = 1,...,n$, $\mathbf{y}_j$ - centers of vortex elements $(\mathbf{l}_j, \Gamma_j)$, $\theta_\varepsilon(|\mathbf{x}|)$ - smoothing function. So the problem of shifting vortex wake can be reduced to matrix-vector multiplication.

APPLYING MOSAIC-SKELETON APPROXIMATIONS

The most obvious way of applying mosaic-skeleton approximations for computing the velocity by formula (1) consists of considering the components of the vector products, applying mosaic-skeleton approximations to them, and performing vector multiplications. In three dimensions, every step of this algorithm involves the approximation of three matrices and six matrix-vector multiplications.

However, using mathematical transformations, we can obtain a velocity expression involving a single scalar matrix approximation and three matrix-vector multiplications.

To calculate the velocity vector $\mathbf{W}$ at receiver points $\mathbf{y}_i = (y_i^1, y_i^2, y_i^3)$, $i = 1,...,m$, for a given set of vortex elements $(\mathbf{l}_j, \Gamma_j)$, $j = 1,...,n$, author proposes the expression $W_i^k = p_i^k \left(A\mathbf{t}^k \right) - q_i^k \left(A\mathbf{f}^k \right) - A\mathbf{h}^k$, where $\mathbf{t}^k = (t_1^k,...,t_N^k)^T$, $\mathbf{f}^k = (f_1^k,...,f_N^k)^T$, $\mathbf{h}^k = (h_1^k,...,h_N^k)^T$, A - matrix with elements $a_{ij} = f_\varepsilon(\mathbf{y}_i, \mathbf{x}_j)$, где $f_\varepsilon(\mathbf{x},\mathbf{y}) = \dfrac{1}{4\pi}\dfrac{\theta_\varepsilon(|\mathbf{x}-\mathbf{y}|)}{|\mathbf{x}-\mathbf{y}|^3}$, $p_i^1 = y_i^2$, $p_i^2 = y_i^3$, $p_i^3 = y_i^1$, $t_j^1 = l_j^3$, $t_j^2 = l_j^1$, $t_j^3 = l_j^2$, $q_i^1 = y_i^3$, $q_i^2 = y_i^1$, $q_i^3 = y_i^2$, $f_j^1 = l_j^2$, $f_j^2 = l_j^3$, $f_j^3 = l_j^1$, h_j^k - components of vector $\left(\mathbf{l}_j \times \mathbf{x}_j \right)$, $k = 1,2,3$. As a result, mosaic-skeleton approximations are applied only to matrix A. When $|\mathbf{x}\text{-}\mathbf{y}| > \varepsilon$ the equality $f_\varepsilon(\mathbf{x},\mathbf{y}) = \dfrac{1}{4\pi}\dfrac{1}{|\mathbf{x}\text{-}\mathbf{y}|^3} \equiv f_0(\mathbf{x},\mathbf{y})$ is done. The asymptotic smoothness of $f_0(\mathbf{x},\mathbf{y})$ was proven in [1].

CALCULATING EXAMPLES

To estimate the efficiency of the numerical algorithms, we computed the unsteady flow past a hemisphere in the direction of the hemisphere axis from the side of an orifice. The results obtained are shown in Fig. 1.

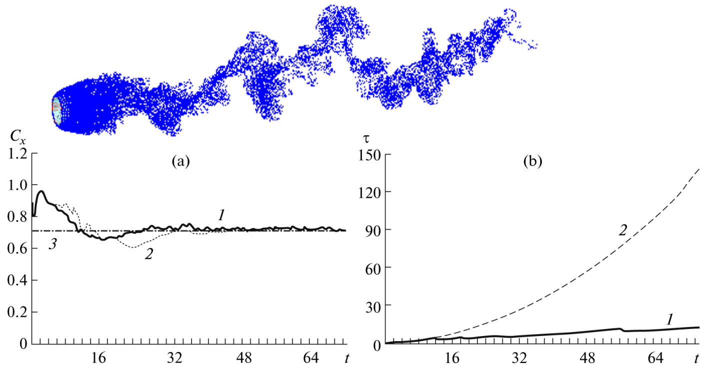

Fig. 1. (a) $C_x(\tau)$ computed by the (*1*) fast and (*2*) direct algorithms and (*3*) the experimental value $C_x = 0.71$. The average computed value is $C_x(\tau) = 0.72$. (b) CPU time per algorithm step for the (*1*) fast and (*2*) direct algorithms.

Fig. 2. depicts velocity fields past group of three cupulas calculated by “straight” and “fast” algorithms and vortex wake shape.

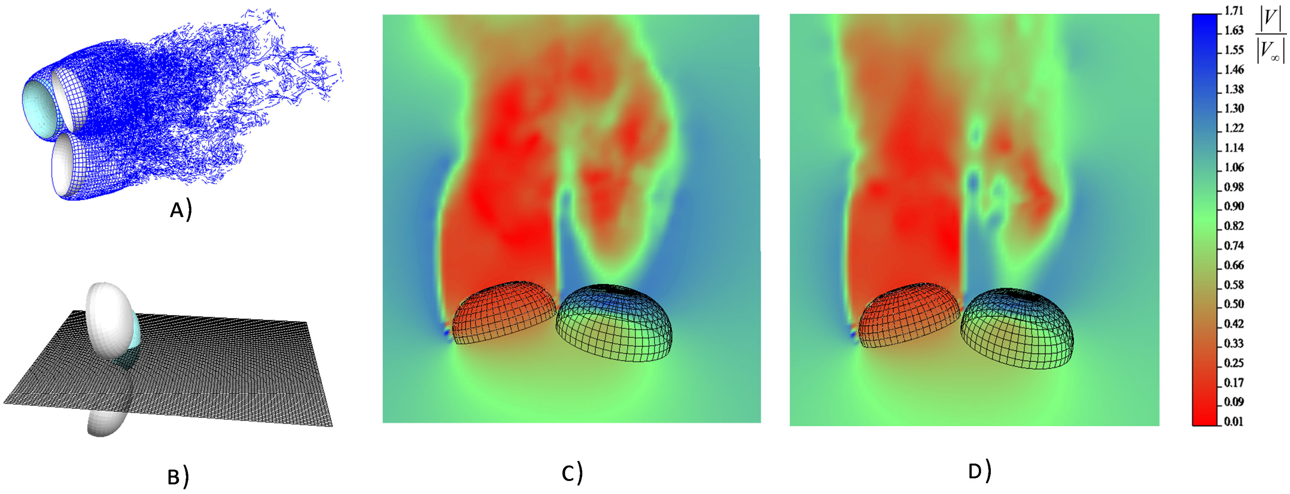

Fig. 2. A) Vortex shape. B) Velocity field calculation section C) Measureless velocity field module distribution calculated with “fast” algorithm D) Measureless velocity field module field distribution calculated with “straight” algorithm

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