NUMERICAL SIMULATION OF THREE-DIMENSIONAL SEPARATED FLOW OVER BODIES WITH SOME VORTEX METHOD

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Summary The numerical simulation of the 3-d unsteady flows of incompressible fluid over bodies with complex geometry with the vortex method is considered in the paper. The vortex wake approximation is based on the Lagrange approach for the inviscid fluid. The surfaces of bodies are simulated with vortex frames. The above method was tested by the examples of simulating unsteady flows over some model objects and to solve actual aerodynamic problems.

INTRODUTION

The method of discrete vortices is one of the most advanced and widespread among vortex methods. The author bases on the vortex methods concept founded by S.M. Belotserkovsky. The great experience was gained over the years of using the method of discrete vortices and the method of closed vortex frames to model unsteady flows of ideal incompressible fluid, to calculate aerodynamic characteristics of air vehicles, research aircraft trails, find aerodynamic characteristics of buildings [1-3]. The main idea of the method is related with simulating the surfaces of bodies in the flow and vortex wakes after them with thin vortex layers and following approximation of these vortex layers with closed vortex lines – vortex frames. However the researchers met a number of drawbacks of these methods such as abnormal deformations of frames which lead to destruction of vortex wave and increasing in time calculation difficulty. To improve the method efficiency author made some advancement. Thus vortex wake was separated into two zones "near" and "far". "Near" wake is approximated with closed vortex frames and "far" wake with isolated vortex segments. The efficiency of the advancements was tested on a number of actual aerodynamic problems.

NUMERICAL METHOD

The 3-D problem of simulating unsteady flow of incompressible fluid past body (group of bodies) is considered. In the described approach it is assumed that the vortex wakes appear on the predefined separation lines. The vortex wake was separated into two zones "near" and "far". It is assumed that the "near" wake is approximated by thin vortex sheet and the "far" wake is a domain with distribution vortices. Non-penetration condition is set at the body surface. It is assumed that body surface and the vortex sheet simulated as surfaces induce velocity field which is represented as the gradient of double-layer potential. The summary fluid velocity field founded in the form:

$$\mathbf{w} = \mathbf{w}_\infty + \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3,$$

$$\mathbf{w}_i = \nabla\varphi_i, \quad \varphi_i(\mathbf{x},t) = \int_{\Sigma_i} \frac{\partial}{\partial \mathbf{n}_y} F(\mathbf{x}-\mathbf{y}) g_i(\mathbf{y},t)\, d\sigma_y,\ i = 1,2,\ F(\mathbf{x}-\mathbf{y}) = \frac{1}{4\pi}\frac{1}{|\mathbf{x}-\mathbf{y}|},$$

$$\mathbf{w}_3(\mathbf{x},t) = \int_\Omega \boldsymbol{\omega}(\mathbf{y},t)\times \mathbf{V}(\mathbf{x}-\mathbf{y})\, d\mathbf{y},\ \mathbf{V}(\mathbf{x}-\mathbf{y}) = -\nabla_x F(\mathbf{x}-\mathbf{y}) = \frac{1}{4\pi}\frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3},$$

where $\mathbf{w}_\infty$ - velocity in infinity, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, Σ_1 is the body surface, $\Sigma_2 = \Sigma_2(t)$ - is the moved surface of the 'near' wake (vortex sheet), Ω - is a vortex domain.
To calculate approximate velocity field the body surface Σ_1 and vortex sheet Σ_2 are approximated with cell systems $\sigma_k^1, k = 1,\ldots,N_1$ and $\sigma_k^2, k = 1,\ldots,N_2$ respectively with constant density of double layer potential at each cell. With this

$$\mathbf{w}_i = \sum_{k=1}^{N_i} g_k^i \mathbf{w}[\sigma_k^i],$$

where $\mathbf{w}[\sigma]$ - velocity induced by cell σ which can be represented by Bio-Savart law:

$$\mathbf{w}[\sigma](\mathbf{x}) = \nabla_x \iint_\sigma \partial/\partial \mathbf{n}_y F(\mathbf{x}-\mathbf{y})\, d\sigma_y = -\int_{\partial S} d\mathbf{l}_y \times \mathbf{V}(\mathbf{x}-\mathbf{y}).$$

Vorticity domain Ω is approximated with the system of isolated vortex segments in such way that

$$\mathbf{w}_3(\mathbf{x}) = \sum_{j=1}^{n} \Gamma_j \mathbf{l}_j \times \mathbf{V}_\varepsilon(\mathbf{x}-\mathbf{y}_j), \quad \mathbf{V}_\varepsilon(\mathbf{x}) = \mathbf{V}(\mathbf{x})\theta_\varepsilon(|\mathbf{x}|),$$

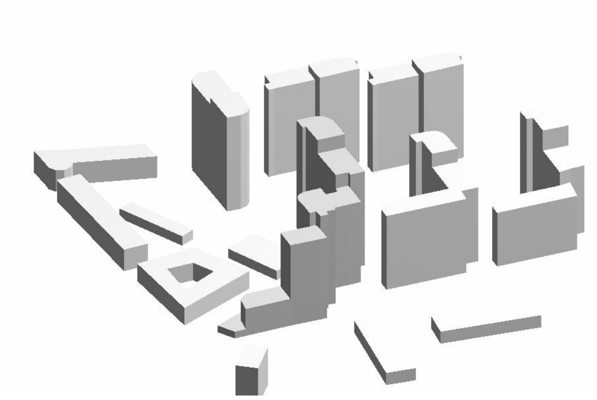
where $\mathbf{l}_j = \mathbf{x}_j^+ - \mathbf{x}_j^-$ - vortex segment with the beginning and the end in the points $\mathbf{x}_j^-$ and $\mathbf{x}_j^+$ respectively, Γ_j - segment intensity, $\mathbf{y}_j$ - centers of vortex elements, $j = 1,\ldots,n$, $\theta_\varepsilon(|\mathbf{x}|)$ - smoothing function [4,5].

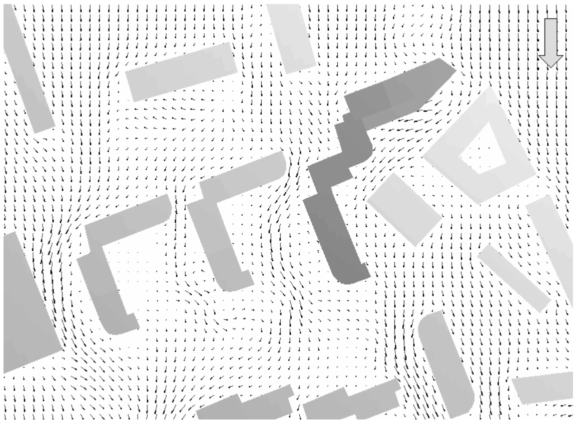
The vortex wake approximation is based on the Lagrange approach for the inviscid fluid. With this the ends of vortex segments from which the vortex frames in "near" zone are composed shift with fluid particles at each step of

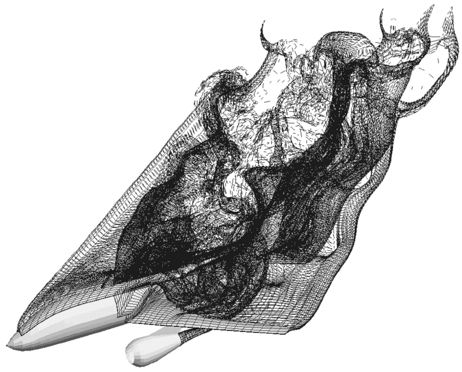
integration in time. After the period when these segments do the predefined number of steps it zone changes to "far". At each step of integration in time the positions of vortex segment ends and intensities of vortex segments are modified in the way that the length of vortex segments do not grow. Segments which had common ends in "near" zone are disrupted when the zone changes to "far" and become isolated.
The unknown intensities of closed vortex frames on the body are found on each step of time integration from the linear equation system that approximate the singular integral equation for unknown double-layer potential density [6]. The above approach makes possible to simulate thin lifting surfaces as well as solid bodies.
To calculate pressure was developed numerical scheme based on using the analog of Cauchy-Lagrange integral for vorticity flows proposed by G. Dynnikova [7].

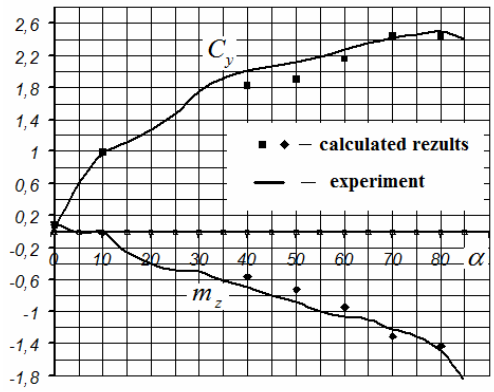
CALCULATION EXAMPLES

The above method is implemented in software system and was tested by the examples of simulating unsteady flows over some model objects and to solve actual problems of calculating aerodynamic characteristics of parachutes, buildings and constructions, airplanes at high angles of attack. In the figures below one can see several examples: calculated velocity field at the ground level around the group of buildings; results of modeling flow past modern aircraft – momentum shape of vortex sheet for the flow past aircraft at the angle of attack $\alpha = 50^{\circ}$ and dependency of normal power coefficient and pitch moment from the angle of attack and comparison of calculated by author results to the experimental data got in TsAGI.









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