SOUND GENERATION IN THE INTERACTION OF TWO ISENTROPIC VORTICES

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Summary The interaction between two isentropic vortices is simulated systematically through solving the two dimensional, unsteady compressible Navier-Stokes equations using the fifth order WENO scheme. Based on the evolution of vorticity and the sound generation, we classified the interaction of two isentropic vortices into four modes. All these four modes can generate sound wave. Where, the forth mode can generate strongest and continuing noise.

INTRODUCTION

Shock waves and vortices are two basic elements of compressible flow. The interaction between two or among more of these elements is a common phenomenon and is very important in many applications such as mixing layers, jets and combustion instability. This interaction can result in the deformation of shock wave and vortices, as well as the generation of sound wave, which play an very important role in aeroacoustic. In particular, because a number of shock waves and vortices coexist in the complicated supersonic turbulence flow, the interaction of a shock wave and vortices can be seen as a simplified model of shock turbulence interaction, which is one of the major sources of noise and has received increasing attention.

There are many studies on shock vortex interaction, which contain experiment, theoretical analysis and direct numerical simulation. A brief summary is given by Inoue and Hattori [2] and Zhang et al. [7]. In our previous studies [8, 9] on the interaction of a shock wave and two vortices, we found that there are two regimes of sound generation. The first regime is linear which corresponds to the shock interaction with two isolated vortices. The second regime is nonlinear, in which the sound wave comes from two processes. One is the vortex coupling, and the second is the interaction between the shock wave and the coupled vortices. Through these studies, we found that the interaction of two or more vortices is very important in sound generation. However, there are fewer studies on the mechanism of sound generation in the interaction. Many studies focused on the dynamics of vorticity, stability, the initial conditions for the onset of merger, especially concerning the critical vortex core radius of two co-rotating vortices[1, 4, 5].

This is an extension of our previous work in Ref. [8, 9]. Using a fifth order WENO scheme [3], we systematically study the interaction between two isentropic vortices through simulating the two-dimensional unsteady Navier-Stokes equation. Our purpose is to study the mechanism of sound generation.

As our physical model, two isentropic vortices with separation distance d are placed in the center of our computational domain ($x_l < x < x_r$, $y_l < y < y_r$). The tangential and radial velocity, pressure and density of a single isentropic vortex [2, 7] are $u_\theta(r) = Mre^{(1-r^2)/2}$, $u_r = 0$, $p(r) = \frac{1}{\gamma}[1 - \frac{\gamma-1}{2}M_v^2 e^{1-r^2}]^{\frac{\gamma}{\gamma-1}}$ and $\rho(r) = [1 - \frac{\gamma-1}{2}M_v^2 e^{1-r^2}]^{\frac{1}{\gamma-1}}$ espectively. where $r = \sqrt{(x-x_v)^2 + (y-y_v)^2}$. (x_v, y_v) is the center of the initial vortex. M_v is the strength of the vortex. $\gamma = 1.4$ is the ratio of specific heats. This isentropic vortex is an exact solution of Euler equation. The initial flow field for the interaction of two vortices is prescribed by the superposition of the flow field produced by each single vortex. After grid convergence study, we use an uniform mesh with the grid density of 6000×6000, which can offer resolved solution including sound waves.

NUMERICAL RESULTS AND DISCUSSION

The interaction of two isentropic vortices has close relation with the rotating directions, the strengths and the initial separation distance of two vortices. Based on the evolution of vorticity field and the generation of sound waves in the interaction, we classified the interaction into four modes.

Mode I: The first mode is the interaction between two counter-rotating vortices with similar strengths, which will result in a new flow structure and generate sound waves if they are close enough. Figure 1 is the evolution of the vorticity and the contours and radial distribution of the sound pressure $\Delta p = \frac{p-p_0}{p_0}$ in the interaction of two counter-rotating vortices with the same strength $M_u = -0.5$ and $M_d = 0.5$. The upper vortex rotates clockwise and the lower vortex rotates in the opposite direction. The initial distance of two vortices is $d = 4$. As can be seen from Figure 1, the initial isentropic vortex has two layers, the inner layer and the outer layer. The sign of vorticity is opposite in the inner layer and the outer layer. The interaction results in a great change in their shapes. The vortex cores are pressed to an approximately elliptical shape from the initially circular shape and they gradually form a vortex dipole, which is advected to the left by the induced velocity. The outer layers move toward the symmetry and gradually separates from the vortex core. They form a weak vortex dipole which moves to the right. Five sound waves with quadrupolar nature are generated at the beginning of the interaction.

Mode II: The second mode is the interaction of two co-rotating vortices without merging, which has folloing features: (1)The interaction evolves into two non-symmetric dipoles; (2)different with the interaction of two counter-rotating vor-

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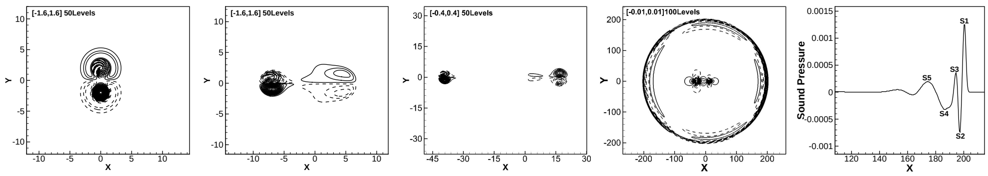

Figure 1. The evolution of vorticity field at typical times (the most left:$t = 0$; the second from left:$t = 60$ and the third from left:$t = 300$) and the contours(the fourth from left) and radial distribution (the most right) of the sound pressure at typical time $t = 200$ in the interaction of two counter-rotating vortices of $M_u = -0.5$, $M_d = 0.5$ and $d = 4$.

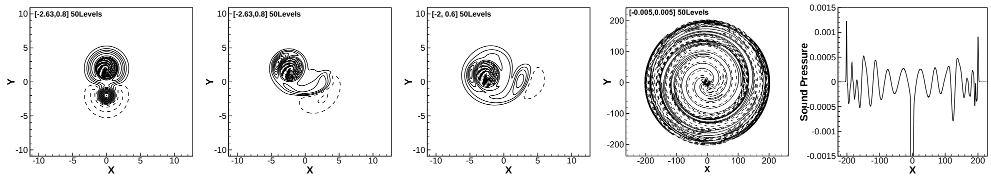

Figure 2. The evolution of vorticity field at typical times (the most left:$t = 0$; the second from left:$t = 60$ and the third from left:$t = 120$) and the contours(the fourth from left) and radial distribution (the most right) of the sound pressure at typical time $t = 200$ in the interaction of two counter-rotating vortices with large difference in their strengths. $M_u = -0.8, M_d = 0.25, d = 4$.

tices, two vortex cores of each vortex dipole result from the same initial vortex. One results from the inner of the initial vortex and the other results from the outer layer;(3)at the late stage interaction, the vorticity of one core is much stronger than the other. As a result, it seems that the weaker one rotates around the stronger one. In this process, very weak sound waves are generated.

Mode III: The third mode is the merging of two co-rotating vortices. In some condition, the interaction of two co-rotating vortices will result in the merger of two vortices [1], which is a key phenomenon in the shear layer and wake flow. Proceeding the merger, the cores of the two vortices move closer together, and the resulting elliptical vortex evolve into a single circular vortex with two arms. In the tail region of each arm, a weak vortex is formed from the out layers of the original vortex. The sign of vorticity of this weak vortex is in opposite with that of the core. The weak vortices rotate in the opposite direction with the stronger one. In the process of vortex merging, stronger noise is generated which is similar to the merging process of two Gaussian vortices [6].

Mode IV: The fourth mode is the interaction of two vortices with large difference in their strengths. From the point view of noise generation, the interaction of two isentropic vortices with large difference in their strength is the most interest. Figure 2 contains the contours of the evolution of vorticity field and the contours and radial distribution of the sound pressure in the interaction between two counter-rotating vortices with large difference in their strengths. Similar to the interaction between two counter-rotating vortices of the same strengths, the interaction evolves into two vortex dipoles. The cores of two initial vortices moves closer and form a stronger vortex dipole. The out layers separate from the initial vortices. They move closer and form a weaker vortex dipole. Because there is much difference in their strengths, both the vortex dipoles are strongly non-symmetric. As a result, the weaker vortex core continuously rotates around the stronger one. At the same time, the weaker vortex dipole rotates around the stronger vortex dipole. In this process, strong continuous noise is generated.

The interaction of two co-rotating vortices with larger difference in their strengths is similar to the above case, which also generates strong continuous noise. The different is that each vortex dipole results from the same initial vortex. Comparing to the noise generation in four modes, we suspects that the strong continuous noise in the fourth mode is generated by the stronger vortex dipole. In this vortex dipole, there is much difference in the strengths of the two vortex cores. The fast rotating movement of the weaker vortex core around the stronger is the key to generate the noise.

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