Vorticity Force on an Impulsively Started Finite Wing at Low Reynolds numbers

Chien C. Chang[*#], Jian-Jhih Lee[*], Cheng-Ta Hsieh[*] & Chin-Chou Chu[*]
[*]*Institute of Applied Mechanics, National Taiwan University, Taipei 106, Taiwan*
[#]*Department of Mathematics & Taida Institute for Mathematical Sciences, Taipei 106, Taiwan*

Abstract. This paper is aimed to examine various contributions to the forces on an impulsively started finite plate from the perspective of a force-element representation. The wing plate has an aspect ratio (AR) between 1 and 3, and is placed at low and high angles of attack (α=5^o, 10^o, 15^o, 30^o, 45^o, and 60^o), while the Reynolds number Re is varied between 100 or 300. The force theory enables us to quantify the contributions to the forces exerted on the plate in terms of all the fluid elements with nonzero vorticity, such as in the tip vortices (TiVs), leading- and trailing-edge vortices (LEV and TEV) as well on the plate surface. The present vorticity force analysis (VFA) was made parallel to the pressure force analysis (PFA) by examining the sectional force contributions along the wing span, but can further extend to include the outer regions (of TiVs). The interplay between the LEV and the TiVs by assessing the relative importance of the transverse as well as the longitudinal vorticity components at various time stages leads to insightful physical explanations of the force mechanisms.

Introduction. Counter-rotating tip vortices are an important aerodynamic feature of wings of finite span due to the pressure difference between the upper and lower wing surfaces. Tip vortices can cause undesirable noise and vibrations as well as induce downwash, which increases the drag force on the wing. In recent years, research in tip vortices has shifted into low-Reynolds-number flows for understanding of animal locomotion with applications to development in micro air vehicles (MAVs) and small unmanned aerial vehicles (UAVs). Because the wing geometry of animal wings and fins typically has a low aspect ratio (LAR), the induced flow is unsteady, and the influence of tip flow is obviously significant. As a model problem, impulsively started LAR plates at low Reynolds number can provide an in-depth research relevant to the design of small aircraft and the aerodynamics of animal locomotion.

In spite of the many important studies on LAR-plate aerodynamics, a quantitative analysis that can identify the primary aerodynamic forces and relate them to individual vortex structures is still lacking. In the literature, there are several useful force theories that shed light on different aspects of hydrodynamic or aerodynamic forces. Circulation theory was an early effort toward predicting lift, which provided insightful relationships between forces and inviscid models in terms of boundary layer separation, vortex shedding, and conservation of circulation. Subsequent studies were meant to provide exact means or theories for hydrodynamic forces through a rigorous analysis of the equation for viscous flow, in particular, ideas that validate a separation of hydrodynamic loadings into potential flow forces and vortex-flow forces. The applied force was deliberately explained as the rate of change of a momentum, defined by an absolutely convergent integral. In this study, we investigate the unsteady mechanisms of a LAR plate with different angle of attack at low Reynolds flows in terms of a diagnostic force theory (Chang 1992; Chang et al. 2008; Hsieh et al, 2009, 2010). The force representation theory for real viscous flow is used to separate potential forces such as added-mass and inertial forces and to distinguish the contributions of individual fluid elements to aerodynamic forces. The theory starts from the D'Alembert theorem that the incompressible potential flow predicts that no force will be exerted on a body if the incident flow is a constant uniform stream. Incompressible potential flow means that there is no single fluid element possessing nonzero vorticity or dilation. It is therefore considered that in a more realistic flow, any fluid element with nonzero vorticity or dilation may be considered a source of the hydrodynamic force.

Results & Discussion. It is shown that there are four regions of distinguishable vortex structures (see Fig. 1 for illustration). Above all, it is noted that the LEV has a separation bubble in the x-y plane for sections near the middle span, and is connected to the tip vortices, the side edges where the roll-up structure contains a separation bubble above the plate in the y-z plane. First, the LEV contributes large positive lift elements, yet near the middle span, there are negative elements inside the separation bubble. Second, the part of each tip vortex above the extended plate contributes large positive lift elements, while along the downstream direction there is a developing region of negative elements inside each separation bubble. Third, we have strong positive elements in the two regions where the LEV and TiVs interlink and rotate against each other. Fourth, the velocity field in the slender region just below the plate contributes substantially negative lift elements. The LEV, TiVs and TEV are interwoven to form a complete vorticity-loop structure. As long as the loop structure is extending downstream the finite wing, its contribution to the lift is quickly diminished. The force-element approach is different from the classical aerodynamic theory. The latter considers the downwash flow of the tip vortices as the cause of reduction in the lift of a finite wing as compared to its 2D counterpart. The former, on the other hand, considers that, as there is no longer a genuinely 2D flow for a finite wing, it is more meaningful to examine directly the contribution from the TiVs to the lift. And indeed, the force element theory shows that the tip vortices are strong sources of lift. In other words, TiVs and LEV as sources of lift (and other force components) are considered on an equal basis; TiVs generated by the side edges, like LEV generated by the front edges, work collaboratively with LEV in contributing lift. In the flow past a finite plate, not only is there spanwise (longitudinal) vorticity, but are also there vorticity components in the other two orthogonal (transverse) directions. In order to examine the three-dimensional effects, we provided two viewpoints: (i) sectional vorticity force distribution (C_{Dv}, C_{Lv}) contrasted with the traditional pressure force distribution (C_{Lp}, C_{Dp}), and (ii) the force credited to orthogonal (transverse) components (ω_x, ω_y, ω_z) contrasted with all of the total force (*CD*, *CL*) credited to all the vorticity components in the entire flow region as well as in the outer regions. It is noted that in the ideal case of two-dimensional flow, there are no outer regions, and CLv' and CLp are identical, and CDv' and CDp are identical. Both viewpoints based on the force element theory show distinguished differences at different AR and angles of attack, and between the early and later stages of flow development. Consider the lift force from viewpoint 1. At the lower AR=1, the sectionwise pressure force (SPF) is always larger than the sectionwise vorticity force (SVF) in the plate region. The deficit of SVF as compared to SPF in the plate region is made up by the positive SVF contribution in the outer regions. The difference between SPF and SVF is more conspicuous at large α. At the large AR=3, the SPF and SVF are usually small in the middle plate region; the discrepancy occurs near the outer edges of the plate. But the situation changes in the case of a large angle of attack (say, α=45^o). In the initial transient, SVF may exceed SPF in the middle of the plate, and becomes much smaller towards the edges of the plate. In the nearly steady state, the difference is small in the middle region of the plate, and becomes large farther from this region. Consider the lift force from viewpoint 2. In the early stages just after the flow is started, the force component due to ω_z dominates other components in the plate region. But the other force components due to ω_x and ω_y have also been developed in the outer region and may exceed the component of ω_z in magnitude. It is found that all the lift components due respectively to ω_x, ω_y and ω_z in the outer regions are positive, and the lift component due to ω_y is invariably the largest at the later time. At the larger AR=3, the lift components due to ω_x and ω_y are typically small in the middle plate region, but may exhibit negative contributions at the outer edges of the plate region. At a lower AR=1 but a larger α, the lift components due to ω_x and ω_y may become substantial in the plate region, though still small compared to that due to ω_z. It is reasonable to consider that ω_z is mainly contributed by the LEV, while ω_x and ω_y are contributed by the TiVs. The temporal variations of the relative importance of force contributions strongly depend on the interplay between the two vortex systems. (i) For the shorter plate (AR=1), the LEV is suppressed by the TiVs in the initial stage, and then develops gradually, extending its vorticity beyond the outer edges. (ii) For the longer plate (AR=3), the TiVs are initially suppressed by the strong LEV, but have more room for development in later stages as the LEV weakens in strength because of diffusion. As a final remark, the VFA for assessing the relative importance of individual force contributions can be pursued for all kinds of finite wings by relating the flow features directly to the aerodynamics forces that have no two-dimensional counterparts, such as those of a delta wing and of many natural flights.

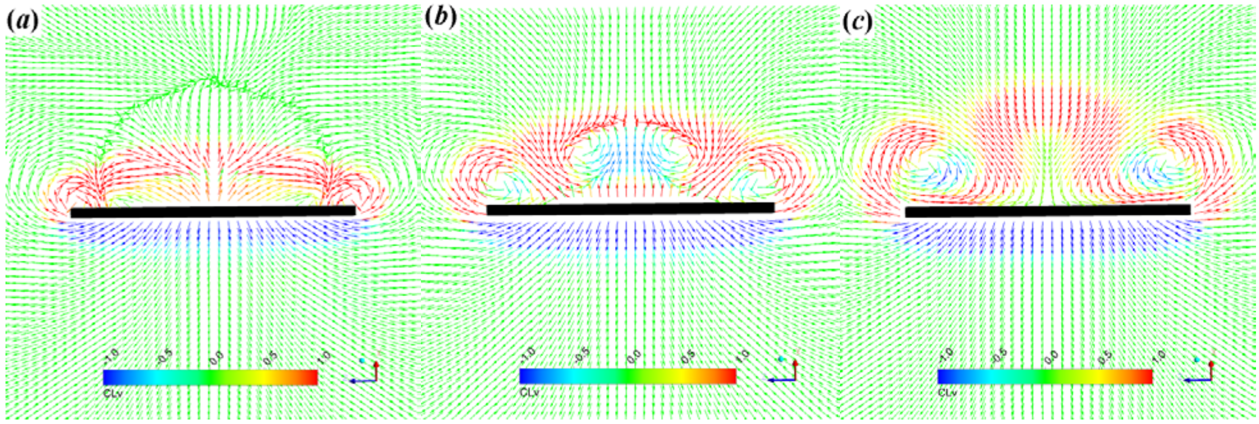

Figure 1. The scales are chosen to be the chord length *L*, and the time *L*/*U*, where *L* is the chord length and *U* the incomin1g flow velocity in the x-direction. The plots show the y-z plane velocity vectors colored by the volume lift elements at various chordwise locations for the AR=1 plate with α=45^o, Re=300 at *t*=1.4: (a) x=0.14 (b) x=0.35 (c) x=0.57. First, the LEV contributes large positive lift elements, yet near the middle span, there are negative elements inside the separation bubble. Second, the part of each tip vortex above the extended plate contributes large positive lift elements, while along the downstream direction there is a developing region of negative elements inside each separation bubble. Third, we have strong positive elements in the two regions where the LEV and TiVs interlink and rotate against each other. Fourth, the velocity field in the slender region just below the plate contributes substantially negative lift elements. Note that at this low Reynolds number, although the contribution from the vorticity within the flow dominates, the surface vorticity contribution is not negligible, typically ranging from 10 to 30% contribution to the lift.

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