NONLINEAR GEOSTROPHIC ADJUSTMENT WITH GYROSCOPIC WAVES

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Summary Nonlinear geostrophic adjustment of rotating fluid confined between upper and lower rigid lids, is examined in the presence of gyroscopic waves. Assuming the Rossby number and the aspect ratio fluid depth/horizontal scale to be small, we demonstrate that the resulting motion is split in a unique way into a quasigeostrophic slow component and an ageostrophic fast component, the slow component being not influenced by the fast one. In barotropic fluid the fast component consists of long gyroscopic waves only, in stratified fluid it includes also internal waves. In both cases the gyroscopic wave component has the form of inertial oscillations modulated by amplitude depending on coordinates and slow time and obeying a linear equation with coefficients depending on the geostrophic part of field.

BAROTROPIC MODEL

We consider the motion in rotating fluid layer, the rotation angular velocity **f** being constant and, generally, not parallel to gravity acceleration. The layer has a constant depth *H* and is confined between upper and lower rigid lids; the Rossby number *Ro* and the aspect ratio *H*/*L* (*L* is the horizontal scale) are assumed to be small and equal in magnitude.

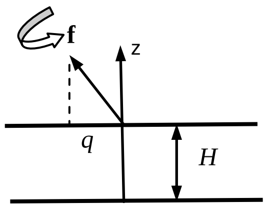

Fig 1. Schematic representation of rotating fluid layer, *q* is the horizontal component of the rotation angular speed **f**

Let the fluid be barotropic, in this case non-dimensional equations of motion can be written in the form:

$$u_t + \delta(uu_x + vu_y + wu_z) - v + \delta qw = -p_x, \quad v_t + \delta(uv_x + vv_y + wv_z) + u = -p_y$$
$$\delta^2 w_t + \delta^3(uw_x + vw_y + ww_z) - \delta qu = -p_z, \quad u_x + v_y + w_z = 0;$$
$$w\big|_{z=0,-1} = 0, \quad (u,v)_{t=0} = (u_I, v_I)(x,y,z),$$

where $\delta = H/L = Ro = U/fL \ll 1$. Obviously, only the gyroscopic waves [1] are possible in this system. Representing all fields in the form of multiple timescale asymptotic expansions (e.g. [2]):

$$(u,v,w,p) = (u_0,v_0,w_0,p_0)(x,y,z,t,T_1,T_2,\ldots) + \delta(u_1,v_1,w_1,p_1)(x,y,z,t,T_1,T_2,\ldots) + \ldots; \quad T_n = \delta^n t,\ n = 1,2,\ldots,$$

we show that the resulting motion is split in a unique way into slow and fast components:

$$(u,v,w,p) = (\bar{u},\bar{v},0,\bar{p}) + (\tilde{u},\tilde{v},\tilde{w},0); \quad (\bar{u},\bar{v}) = \int_{-1}^{0}(u,v)dz, \quad \int_{-1}^{0}(\tilde{u},\tilde{v})dz = 0\,.$$

The slow component $(\bar{u},\bar{v},0,\bar{p})$ is not influenced by the fast component and does not depend on the fluid depth, it is quasigeostrophic and obeys the quasigeostrophic potential vorticity equation coinciding here with 2D fluid dynamics equation:

$$\bar{u} = \bar{u}(x,y,T_1) = -\psi_y, \quad \bar{v} = \bar{v}(x,y,T_1) = \psi_x, \quad \frac{\partial \Delta\psi}{\partial T_1} + J(\psi,\Delta\psi) = 0\,.$$

The fast component consists of long gyroscopic waves and has the form of inertial oscillations modulated by amplitude depending on coordinates and slow time and obeying equation with coefficients depending on ψ, i.e. the fast component is coupled to the slow one:

$$\tilde{u} + i\tilde{v} = A(x,y,z,T_1)e^{-it}, \quad \tilde{w} = -\frac{1}{2}e^{-it}\int_{-1}^{z}(A_x - iA_y)dz + c.c.\,,$$

$$\frac{\partial A}{\partial T_1} + J(\psi, A) + \frac{i}{2}\Delta\psi A + iq\left(\int_{-1}^{z} Adz + \int_{-1}^{0} zAdz\right)_y = 0, \quad \int_{-1}^{0} Adz = 0\,.$$

The term proportional q in the amplitude equation is due to the "non-traditional" terms in the equations of motion related to the horizontal component of the Coriolis force.

STRATIFIED FLUID

In stratified fluid the gyroscopic waves are possible only if $N_{\min} < f$, where N is the Brunt-Vaissalla frequency, i.e. if there exist domains with sufficiently weak stratification [3]. Here we consider two-layer model in which the fluid density is continuous, but the buoyancy frequency N is constant in the upper layer and zero in the lower one (see Fig. 2). In this case the initial state also is split into slow quasigeostrophic and fast ageostrophic components. The slow component is 3D in the stratified upper layer and does not depend on the depth in the homogeneous lower layer. The fast component is a superposition of internal and gyroscopic waves. The internal waves are dynamically "insignificant" because they quickly run away to infinity, and the gyroscopic waves again have the form of modulated inertial oscillation like in barotropic fluid but in stratified case they are confined to the homogeneous lower layer. Equation for the oscillation amplitude is practically the same as in barotropic case.

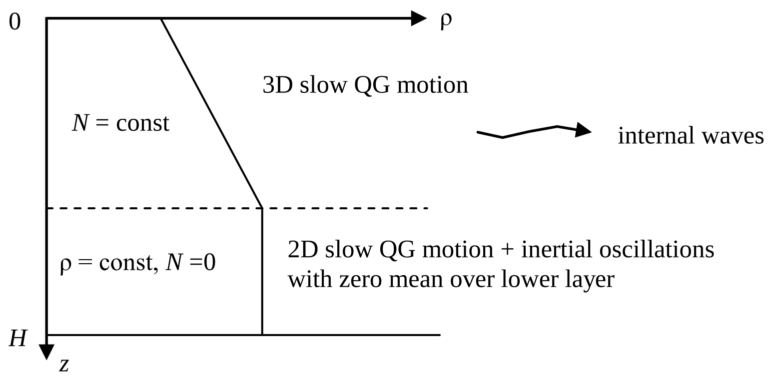

Fig.2 Schematic representation of the quasigeostrophic adjustment in stratified fluid with gyroscopic waves

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References

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