PASSIVE SCALAR ADVECTION IN THE VICINITY OF ELLIPSOIDAL VORTEX IN A DEFORMATION FLOW

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Summary
Quasi-geostrophic dynamics of an ellipsoidal vortex embedded in a nonuniform flow is studied in the approximation of the infinitely deep rotating ocean with a constant buoyancy frequency. The vortex core is an ellipsoid with a constant vorticity different from the background vorticity value. The core is shown to move along with the flow and to deform under the effect of it. The localized regimes of the core motion are analyzed. The mechanisms of fluid particle trajectory chaotization are revealed; in particular, it is shown that, owing to the double periodicity of the core motion, all the nonlinear resonances appear as pairs of two resonance islands with the same winding number.

Quasi-geostrophic dynamics of an ellipsoidal vortex embedded in a nonuniform flow is studied in the approximation of the infinitely deep rotating ocean with a constant buoyancy frequency. The vortex core is an ellipsoid with a constant vorticity different from the background vorticity value. The background flow is assumed to be purely horizontal and linearly depending on coordinates:

$$u_b = ex - \gamma y, \quad v_b = \gamma x - ey; \tag{1}$$

the parameters e, γ are constant. Equation which determines the boundary evolution can be written in the form:

$$F(x, y, z, t) = \frac{\tilde{x}^2}{a^2(t)} + \frac{\tilde{y}^2}{b^2(t)} + \frac{\tilde{z}^2}{c^2} - 1 = 0, \tag{2}$$

where the coordinates $(\tilde{x}, \tilde{y})$ are related to the coordinates (x, y) by equations

$$\tilde{x} = (x - x_0)\cos\theta(t) + (y - y_0)\sin\theta(t),$$
$$\tilde{y} = -(x - x_0)\sin\theta(t) + (y - y_0)\cos\theta(t).$$

We also need to calculate the stream function. It is convenient to write the stream function as a sum $\psi = \psi_b + \psi_V$, where ψ_V is the perturbation due to the vortex. These components satisfy the equations

$$\Delta\psi_b = 2\gamma, \quad \Delta\psi_V = \begin{cases} 0, & \mathbf{r} \notin V, \\ 2g, & \mathbf{r} \in V. \end{cases} \tag{3}$$

For the vortex part we have (see Zhmur and Pankratov, 1989; Meacham *et al.*, 1994)

$$\psi_V(\tilde{x}, \tilde{y}, \eta) = -\frac{1}{2} gab\tilde{c} \int\limits_{\lambda(\tilde{x},\tilde{y},\eta)}^{\infty} \left(1 - \frac{\tilde{x}^2}{a^2+\mu} - \frac{\tilde{y}^2}{b^2+\mu} - \frac{\eta^2}{\tilde{c}^2+\mu}\right)\frac{d\mu}{\sqrt{\Delta(\mu)}}. \tag{4}$$

The vortex core evolution satisfied to two equations for ε, θ (Zhmur and Pankratov, 1989)

$$\dot{\varepsilon} = 2e\varepsilon\cos 2\theta, \quad \dot{\theta} = \Omega(\varepsilon) + \gamma - e\frac{\varepsilon^2+1}{\varepsilon^2-1}\sin 2\theta, \tag{5}$$

where $\xi = \frac{a}{b} + \frac{b}{a} = \varepsilon + \frac{1}{\varepsilon}$. The integration of (5) give us (Zhmur and Pankratov, 1989):

$$\sin 2\theta(\varepsilon) = \frac{\varepsilon}{\varepsilon^2 - 1}\left\{\frac{\varepsilon_0^2 - 1}{\varepsilon_0}\sin 2\theta_0 + \frac{1}{e}\int\limits_{\varepsilon_0}^{\varepsilon}\frac{\tau^2-1}{\tau^2}\Omega(\tau)d\tau + \frac{\gamma}{e}\left(\varepsilon + \frac{1}{\varepsilon} - \varepsilon_0 - \frac{1}{\varepsilon_0}\right)\right\}, \tag{6}$$

where θ_0, ε_0 are some initial values. The integral in (6) may be expressed in a symmetric form of elliptic integral (Carlson and Gustafson, 1993; Dritschel *et al.*, 2004).

The core is shown from (6) to move along with the flow and to deform under the effect of it. Regimes of the core's behavior depend on the flow characteristics and the initial values of the vortex parameters (the shape and the orientation relative to the flow). These regimes are (i) rotation (along with the eccentricity oscillation), (ii) oscillation about one of the two specific directions (along with the eccentricity oscillation), and (iii) infinite horizontal elongation of the core. On the 1 the regions for different regimes of vortex core motion on (gK, e)-plane are shown.

The localized regimes (rotation and oscillation) of the core motion are analyzed. It is shown, that zones of the water mass capturing can appear in the induced velocity field. The mechanisms of fluid particle trajectory chaotization are revealed, in particular, it is shown, that, owing to the double periodicity of the core motion, all the nonlinear resonances appear as pairs of two resonance islands with the same winding number. On the **??** was shown the example of phase portrait and Puancare section of fluid particle motion in induced velocity field.

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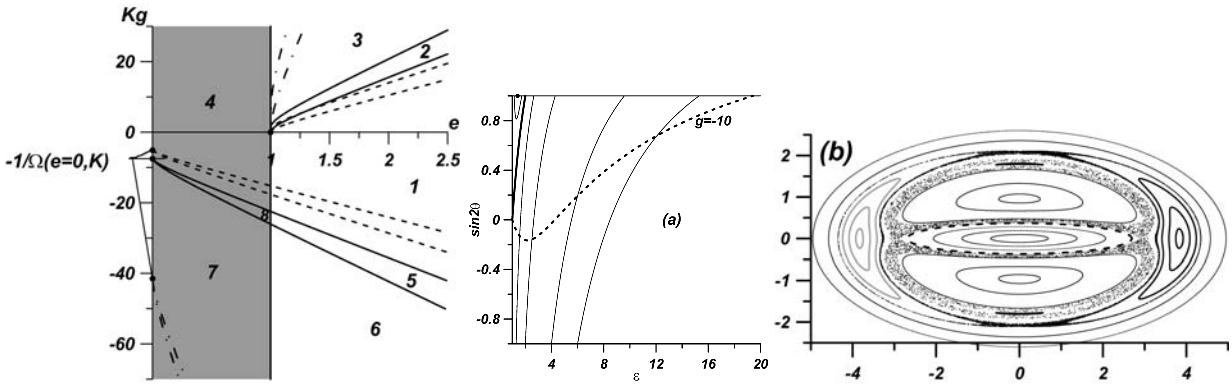

Figure 1. (a) - The regions for different regimes of vortex core motion on (gK, e)-plane. The solid lines correspond to $K = 1$. The dashed lines correspond to $K = 0.01$, and the dot-dashed lines, to $K = 10$. The white background corresponds to $e > 1$ with motion types of 1-3,5,6; the gray background corresponds to $e \leq 1$ with motion types of 4,7,8. (b) - Phase trajectories on the plane $(\varepsilon, \sin 2\theta)$, when $K = 1, e = 0.5 < 1 \quad g = -16.7$ ($g = -10$ - dotted line), regime 8. (c) - fluid particle trajectories in a velocity field induced by a vortex core located near the elliptic point ($g = -10$,$\varepsilon_0 = 7.46$,$\sin 2\theta_0 = 1$).

CONCLUSION

Though the deformation flow we considered is stationary, the motion of passive fluid particles is described by a nonintegrable 1.5 degrees of freedom dynamical system. In this case, the additional "half" degree of freedom appears because the motion of the vortices about their stationary positions is periodic. The periodic motion of the vortices plays the role of a periodic perturbation for the system describing the passive particle dynamics. Therefore chaotic advection of passive fluid particles in the vicinity of the vortex pair can occur.

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