TIME CORRELATIONS IN COMPRESSIBLE ISOTROPIC TURBULENCE

Dong Li*, Guo-Wei He*a) & Xing Zhang*
*State Key Laboratory of Nonlinear Mechanics, Institute of Mechanics, Chinese Academy of Sciences, Beijing, 100190, China

Summary Time correlations of fluctuations in velocity, dilatation and pressure are calculated using direct numerical simulation. It is observed that the correlations decay with oscillations. A model is proposed for time correlation. It combines the classical sweeping hypothesis and the wave propagation model. This model is found to fit well with the time correlations. All curves of time correlations collapse into single one when rescaling the separation using the present model.

INTRODUCTION

Time correlation is essential to turbulence statistic theory with critical application to aeroacoustics. The models for time correlations in incompressible turbulence have been developed, such as Taylor frozen flow model[1], Kraichnan sweeping model[2] and recently developed non-frozen flow mode[3]. However, there are few models available for compressible turbulence. In this study, we will develop the model for dilatational part in compressible turbulence.
Taylor's frozen turbulence hypothesis[1] assumes that the turbulent structures are advected by mean flow without changes. His assumption relates the spatial and time variations by mean velocity U. It agrees well with experiments and the concept that the spatial pattern of two points can be replaced by the temporal response at one point is quite useful in turbulence measuring. The convection velocity of a single Fourier mode is $u=\omega/k$. Taylor's approximation uses mean velocity U to remove the dependence on frequency and wavenumber. It is valid if the mean shear is small and turbulence level is low. Taylor's hypothesis matches well for the divergence-free part of the flow, but does not work on dilatational part. Lee[4] gives a propagation model based on Taylor's hypothesis using the idea that dilatational part obeys the wave equation.
The random sweeping hypothesis[2] proposed by Kraichnan and Tennekes suggests that the small eddies are convected by energy-containing eddies in isotropic turbulence. This approximation describes the decorrelation process of fluctuating velocities in time and is extended to fluctuating pressure by Yao[5] in incompressible isotropic turbulence.
DNS of stationary and decaying compressible isotropic turbulence by solving the fully compressible N-S equations are performed. The decorrelation patterns of dilatational part are analyzed using a model that combines the propagation model based on Taylor's hypothesis and the sweeping approximation.

METHODS AND DISSCUSIONS

The velocity time correlation coefficient is defined as

$$c_u(k,\tau)=\frac{\langle u_i(\mathbf{k},t)u_i(-\mathbf{k},t+\tau)\rangle}{\langle u_i(\mathbf{k},t)u_i(-\mathbf{k},t)\rangle}. \tag{1.1}$$

Following the random sweeping hypothesis, the velocity time correlation coefficient can be expressed as

$$c_u(k,\tau)=\exp\left(-\frac{1}{2}v^2k^2\tau^2\right). \tag{1.2}$$

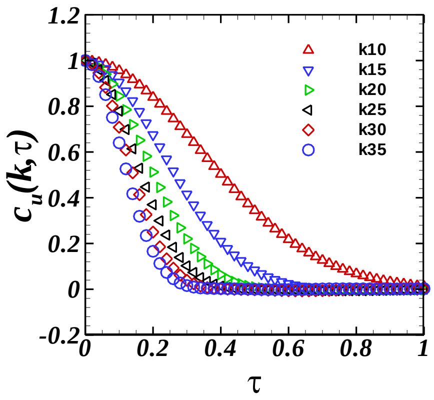

Fig.1 Normalized time correlations of incompressible velocity vs time separations.

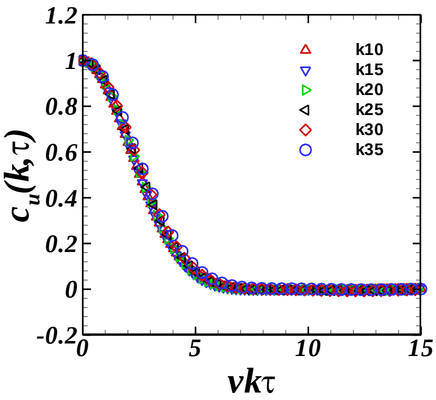

Fig.2 Normalized time correlations of incompressible velocity vs time separations scaled by $(vk)^{-1}$.

a) Corresponding author. Email: hgw@lnm.imech.ac.cn

This model predicts a universal form for time correlations of velocity modes and agrees well with our simulation in Fig.1 and Fig.2 for divergence-free part of velocity in stationary isotropic turbulence.
Fig.3 and Fig.4 show the decorrelation pattern of fluctuating dilatation in stationary isotropic turbulence. The coefficient decays with oscillations and decreases faster than that of incompressible velocity and all correlation curves collapse into single one if time separations are scaled by the sweeping time scale $(vk)^{-1}$.
The linear wave propagation model[4] proposed by Lee takes dilatation fluctuation as wave without source and the space-time correlation of dilatation can be expressed as

$$R(\xi,\tau)=\int_0^{\infty}E^{\theta}(k)\cos(k\bar{a}\tau)\frac{\sin\left[k(\xi-U_1\tau)\right]}{k(\xi-U_1\tau)}dk. \qquad (1.3)$$

The wave model performs well in the case of large mean velocity if the time separation is short. But it does not agree with simulation result if the time separation is long.
The issue is how to indicate the influence of turbulence eddies. We also use wave equation as the governing equation of dilatational part. The difference is that we represent the influence of eddies to dilatational part using the random sweeping hypothesis. The source term of wave equation is dominated by energy-containing eddies, thus the dilatational part of turbulence is represented as primary waves plus new waves injected by eddies from flow field. The solution gives an attenuation term to the correlation coefficient.
Fig.5 gives the model prediction of normalized time correlations of dilatation modes vs time separations scaled by $(vk)^{-1}$. The shape of current model predicted agrees well with our simulation result.
This model also works well in decaying isotropic turbulence where the correlation coefficient of dilatation fluctuation is quite like oscillating attenuated wave.

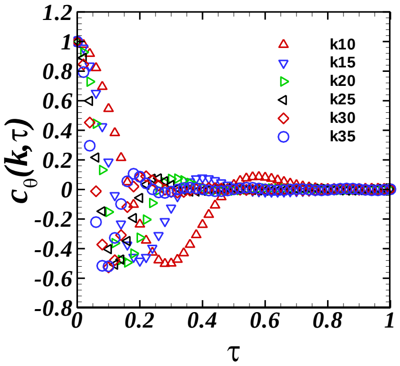

Fig.3 Normalized time correlations of dilatation vs time separations

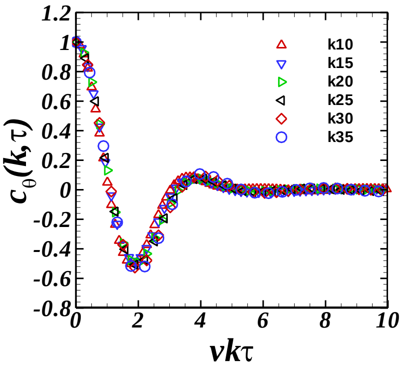

Fig.4 Normalized time correlations of dilatation vs time separations scaled by $(vk)^{-1}$.

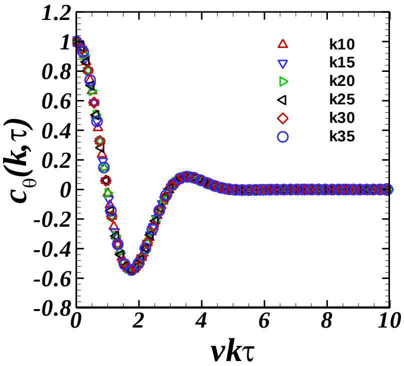

Fig.5 Model prediction of normalized time correlations of dilatation vs time separations scaled by $(vk)^{-1}$.

CONCLUSIONS

DNS of stationary and decaying compressible isotropic turbulence are performed to investigate the time correlations of dilatational part. The correlation coefficients of dilatational part are quite different from that of divergence-free part. A model is developed for time correlation in compressible isotropic turbulence which combines the classic sweeping model and the wave model. The model is validated using the DNS data and used to explain the observed correlation patterns of dilatational part.

References

[1] Taylor G.I.: The spectrum of turbulence. *Proc. R. Soc. Lond. A* **164**: 476-490, 1938.
[2] Kraichnan R.H.: The structure of isotropic turbulence at very high Reynolds numbers. *J. Fluid Mech* **5**(4): 497-543, 1959.
[3] He. G. W. and Zhang. J. B.: Elliptic model for space-time correlations in turbulent shear flows. *Physical Review E* 73: 055303(R), 2006.
[4] Lee S., Lele S.K., Moin P.: Simulation of spatially evolving turbulence and the applicability of Taylor's hypothesis in compressible flow. *Phys. Fluids A* **4**(7): 1521-1530, 1992.
[5] Yao H.D., He G.W., Wang M., Zhang X.: Time correlations of pressure in isotropic turbulence. *Phys. Fluids* **20**(2): 025105, 2008.