A NON-FROZEN FLOW MODEL FOR PRESSURE SPACE-TIME CORRELATIONS IN TURBULENCE SHEAR FLOWS

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Summary We propose a non-frozen flow model of pressure space-time correlation in turbulence shear flows. This model connects the space correlation with the time correlation via a nonlinear transformation. It is a higher-order approximation to the correlations while Taylor's frozen flow model is a linear approximation.

1. Introduction

A space-time correlation is the minimal quantity to describe spatial-temporal statistics of turbulent flows. Taylor's frozen-flow hypothesis[1] is well-known model for space-time correlations. However, Taylor's model has many limitations such as low turbulence intensity, weak shear and local isotropy[2]. Our previous work[3] points out that the frozen flow assumption in Taylor's model implies the first approximation to iso-correlations contours and thus propose a second approximation. This leads to the elliptic model[4] for space-time correlations of velocity fluctuations with the experimental[5] and numerical validations. In the present study, we will develop a model of high-order approximation for pressure fluctuation, which could include the non-frozen flow effects.

The space-time correlation of pressure fluctuations is important in aero- and hydro-acoustic applications. It is defined as

$$R(\tau,r)=\frac{\langle p(x+r,y,z,t+\tau)\,p(x,y,z,t)\rangle}{\langle p^2(x,y,z,t)\rangle},$$

where p is the pressure fluctuation, r the space separation in streamwise direction and τ the time separation. Due to the stationary flows, the correlation is independent of spatial location and time.

2. A non-frozen flow model of pressure space-time correlation

The data from direct numerical simulation of turbulent channel flows is used to study the iso-correlation contours for pressure fluctuations. The observation results in the similarity assumptions: the iso-contours share a preference direction with modulated aspect ratios. We use the assumptions to find the mathematic formula for the iso-contours and derive a model as follows

$$R(\tau,r)=R(0,r^*)\quad\text{where}\quad r^*(r,\tau)=\left(1+b\tau^c\right)^{-1}\cdot\sqrt{\left(r-U\tau-a\tau^2\right)^2+V^2\tau^2}\,.$$

Here R is the space-time correlation, U gives the convection velocity. a is the convection acceleration which makes the preference direction bending towards the time separation axis. V is the sweeping velocity which controls the width of the contour. $\left(1+b\tau^c\right)^{-1}$ is the additional decaying factor which makes the correlation decays faster as time separation increases. b and c in the model are positive.

3. Numerical verification

The non-frozen flow model will be verified by the data from turbulent channel flows at viscous Reynolds number $\mathrm{Re}_\tau=180$. The channel is $24\pi\delta\times2\delta\times4\pi\delta$ in streamwise, wall normal and spanwise direction, where δ is half channel width. The computation domain is long enough for both velocity and pressure space-time correlation decays to almost 0, even on the channel centre. The mesh points are $2048\times129\times384$. Spectral method is used for spatial discretization, and time discretization using multi-step method[6]. The divergence free is enforced by the method proposed by Schumann[7]. Pressure space-time correlation is averaged by 1000 time steps and in both streamwise and spanwise direction.

Figure 1 shows the pressure space-time correlations, against space separation for different time delays. As τ increase the peak of the space-time correlation moves towards the larger space separation direction, which indicates a convection velocity. Convection velocity direction is the preference direction of space-time correlation in which the correlation decays slowly. Figure 2 shows the space-time correlation against the normalized separations using Taylor's hypothesis. The velocity used in the hypothesis is the convection velocity. They couldn't collapse together. The peaks of lines with $\tau\neq0$ is lower than unity. The peak position does not rests at $r-U\tau=0$ which indicates a non-uniform convection velocity at different time separation. And the correlation decays slower as τ increasing. In the non-frozen flow model,

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convection acceleration a gives a non-uniform convection velocity, and the factor $\left(1+b\tau^{c}\right)^{-1}$ gives a different decay rates with different τ . The model is applied to pressure correlation of channel flow. Parameters of the model are extracted from the data by minimizing the difference between the points of space-time correlation and the points predicted by the model. Correlation in the log-law region ($y^{+}=95.15$) is taken as an example. The parameters are listed in table 1 and the results are shown in Figure 3. Convection velocity and acceleration from optimization procedure and definition are close to each other. The exponent of time separation in the decaying factor is almost 1. From this observation, it can further be simplified as $\left(1+b\tau\right)^{-1}$, and b has a dimension of one over time. The model is checked on wall of channel, viscous sublayer and buffer layer. The model can collapse to space correlation on all three layers, but on channel centre the model diverges from the space correlation. This discrepancy might be caused by vanishing mean shear rates on the channel centre. The correlation at Figure3 is truncated at 0.1 as the points less than 0.1 fluctuates around 0 and these parts contribute less to the fluctuation energy compared to the larger correlations.

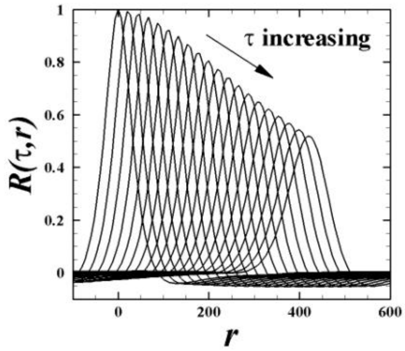

Figure 1. Pressure space-time correlation on $y^{+}=95.15$

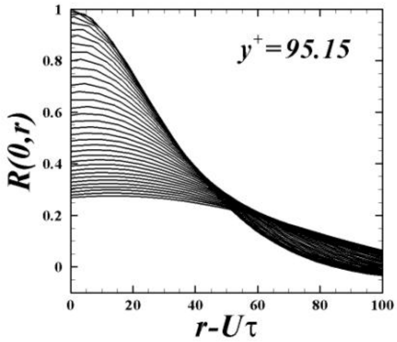

Figure 2. Space-time correlation against the normalized separations using Taylor's hypothesis

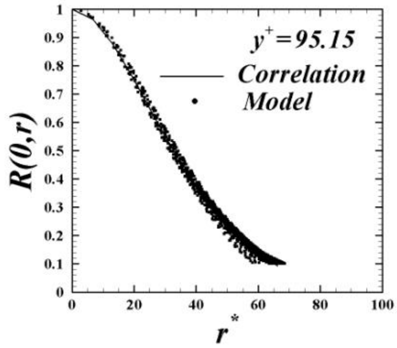

Figure 3. Space correlation from space-time correlation using the non-frozen flow model

y^{+}	Optimized					Definition	
	U	a	V^{2}	b	c	U	a
95.15	15.69	0.0041	2.93	0.012	1.057	16.06	0.00065

Table 1. Parameters of the model optimized are the parameters calculated by the minimization procedure. Parameters in the definition column are from the definition of preference direction.

4. Conclusions

We develop a non-frozen flow model for space-time correlations of pressure fluctuations. It is a higher-order approximation to iso-correlation contours in comparison with the well-known Taylor's frozen flow model. This model is validated by the DNS of turbulent channel flows. It will be further used to study turbulence-generated noise.

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