ENERGY AND HELICITY CASCADES IN ROTATING TURBULENCE

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Summary The cascade processes in homogeneous stationary turbulence simultaneously excited by a helical force and affected by rotation are considered. Numerical simulations at very large Reynolds numbers are performed using the shell model based on decomposition of helicity into eigenmodes. It shown that the energy spectrum forms two inertial ranges with "-2" and "-5/3" slopes. Energy and helicity spectral fluxes keep constant with within these well pronounced inertial ranges. The helicity is transferred as a passive admixture, its dissipation appears at the same scales as the energy, though the helicity transfer dynamics on the inertial range are different on the large and small scales. We reconsider some results about helical rotating turbulence obtained in DNS which may be caused by limited spectral resolution.

MOTIVATION

Classical (Kolmogorov) scenario of a turbulent flow of an incompressible, isothermal fluid can be changed considerably under the influence of the general rotation and the helicity. Rotation leads to a decrease in the rate of energy dissipation associated with the suppression of the cascade energy transfer from larger to smaller scales [7]. The assumption of proportionality of the interaction characteristic time at given scale to characteristic rotation time of system provides a simple phenomenological relations for a large-scale part of the spectrum [8]. The law "-2" was confirmed by the numerical (eg, [6]) and experimental [1] studies.

Total helicity is an integral of motion in an ideal three-dimensional hydrodynamics of an incompressible fluid. In most real flows the helicity is zero, which corresponds to nonhelical conventional turbulence. In the special case of helical turbulence it is discussed a variety of scenarios, most fascinating is the inverse cascade of energy with a direct cascade of energy and helicity [4]. The global rotation and the helicity is closely related. The Coriolis force together with the field gradient (density, pressure, intensity of turbulent fluctuations) may lead to the generation of helicity. Numerical simulation of forced helical turbulence with rotation, carried out in [3], confirmed that the presence of rotation and cascade of helicity significantly affects the dynamics of small-scale turbulence. However, we note that the numerical experiment is not possible to reproduce the well developed inertial range. Therefore, the behavior of rotating helical turbulence at very large Reynolds numbers is still the point of interest.

HELICAL SHELL MODELS WITH THE CORIOLIS FORCE

The main idea of the shell model is to describe the statistics of homogeneous and isotropic turbulence in spectral space using a simple set of ordinary differential equations. The shells being logarithmically spaced, and taking one unknown per shell, such models allow to investigate some turbulence properties at a numerical cost drastically lower than a direct numerical simulation. It is then possible to reach Reynolds numbers defined at scale k_F by $Re = C_E^{1/2}\varepsilon^{1/3}k_F^{-4/3}\nu^{-1}$, as large as 10^7 as will be done below. Shell models with proper definition of helicity were suggested by [9]. Later the helical shell model was developed [2] based on the helical Fourier modes decomposition. This approach leaded to a single formula for four independent models:

$$d_t u_n^{\pm} = Q_n^{\pm} - \nu k_n^2 u_n^{\pm} + f_n^{\pm} + \imath\Omega u_n^{\pm}, \tag{1}$$

with

$$Q_n^{\pm} = \imath k_n\left[(s_1\lambda - s_2\lambda^2)u_{n+1}^{\pm s_1}u_{n+2}^{\pm s_2} + (s_2\lambda - \lambda^{-1})\,u_{n-1}^{\pm\, s_1}u_{n+1}^{\pm s_2 s_1} + (\lambda^{-2} - s_1\lambda^{-1})\,u_{n-2}^{\pm s_2}u_{n-1}^{\pm s_1 s_2}\right]^*, \tag{2}$$

where each model is retrieved for one particular choice of (s_1, s_2) with $s_1,\ s_2 = \pm 1$. In (2) the parameter λ is the logarithmic shell spacing and the wave number is defined as $k_n = \lambda^n$. We use a forcing f_n only in the first shell ($k_F = 1$) with a random phase. The last term in (2) corresponds to the Coriolis force. The shell model (2) is designed such that total energy E and helicity H are conserved in the absence of forcing and viscosity ν. They are defined by

$$E = \sum_{n=1}^{N} E_n, \qquad H = \sum_{n=1}^{N} H_n \tag{3}$$

where N is the number of shells in the model. The energy E_n and helicity H_n in shell n are defined

$$E_n = E_n^+ + E_n^-, \quad E_n^{\pm} = \frac{1}{2}|u_n^{\pm}|^2, \tag{4}$$

$$H_n = H_n^+ + H_n^-, \quad H_n^{\pm} = \pm\frac{1}{2}k_n|u_n^{\pm}|^2 \tag{5}$$

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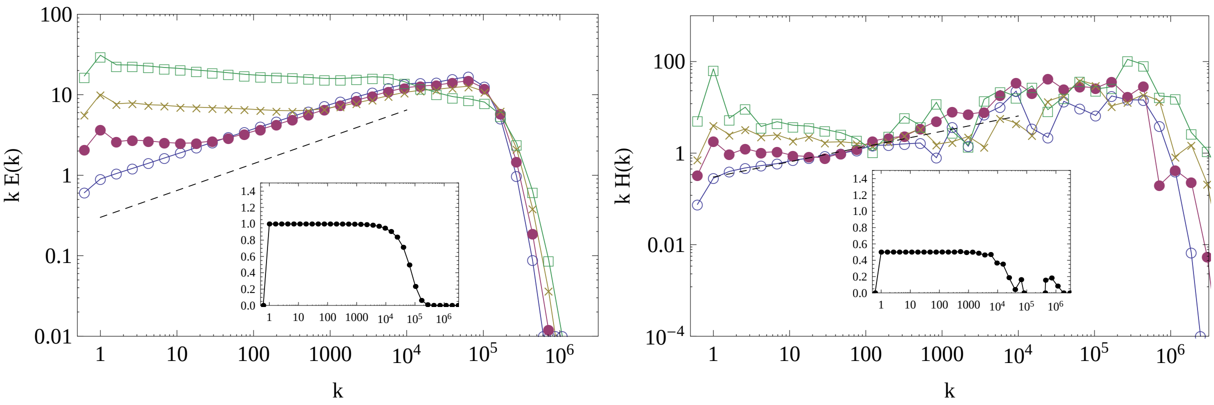

Figure 1. Normalized energy spectra (a) and helicity spectra (b) for different Ω: 0 (open circles), 10 (closed circles), 10^2 (crosses), 10^3 (boxes). Inset shows the spectral flux.

RESULTS OF NUMERICAL SIMULATIONS

Figure 1 shows normalized energy and helicity spectra for different rotation rate Ω. Energy spectrum demonstrates transition from slope "-2" to "-5/3". Helicity spectrum is highly fluctuating at small scales but obeys the same spectral indexes. The inertial range can be observed in wide range of scales, more than three order of magnitude. More statistical properties of rotating helical turbulence and its comparison with results of intensive DNS [4, 5] will be presented in a full paper and presentation.

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