EXPERIMENTAL INVESTIGATION OF HYPERSONIC BOUNDARY LAYER INSTABILITY AND TRANSITION BY WAVELET TRANSFORM

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Summary The instability of hypersonic boundary layer on a sharp cone is investigated in a hypersonic wind tunnel. The time signals of fluctuating surface-thermal-flux are measured by Pt-thin-film thermocouple temperature sensors mounted at 28 stations on the cone surface along the streamwise direction to investigate the development of the unstable disturbances. Wavelet transform is employed to obtain the multi-scale characteristics of fluctuation both in temporal and spectrum space.

INTRODUCTION

Hypersonic flow instability and transition from laminar to turbulence are critical aspects in the design of hypersonic vehicles as they are characterized by marked increases in surface heat flux and skin friction [1]. The accurate prediction of the laminar-turbulence transition position is the key in designing hypersonic vehicles. The laminar-turbulence transition process is characterized by the development of instability modes in boundary layer. Unlike the case in a subsonic flow, at hypersonic velocities, a new acoustic type of instability (the so-called second mode instability) [2] arises in addition to the vortex instability mode (the first mode instability).

Wavelet transform

Wavelet transform convolutes time series signals with an analytic function named wavelet at a definite time and scale by means of dilations and translations of mother wavelet. It provides a two-dimensional unfolding of one-dimensional signals resolving both on spatial and temporal dimensions as independent variables. The continuous wavelet transform of signal $s(t)$ with respect to a family of wavelet $\psi_{ab}(t)$ is defined as their scalar product by:

$$\psi_s(a,b)=\int_{-\infty}^{+\infty} s(t)\overline{\psi_{ab}(t)}\mathrm{d}t \tag{1}$$

Wavelet cross-spectrum

The wavelet cross-spectrum is defined by wavelet coefficients $\psi_s(a_1,\tau)$ and $\psi_s(a_2,\tau)$ at two scales a_1 and a_2:

$$B_s(a_1,a_2)=\int_T \psi_s(a_1,\tau)\overline{\psi_s(a_2,\tau)}\mathrm{d}\tau \tag{2}$$

The cross-spectrum [9][10] depends heavily on the behavior of the signals at two frequencies $f_1=c/a_1$ and $f_2=c/a_2$.

Bicoherence spectrum analysis

Since the modulation of Mack modes during the nonlinear evolution may be important in a hypersonic transition, the wavelet bispectrum[3] is a suitable technique for the dual time-frequency analysis of the wavelet transform through the bispectrum. The wavelet bispectrum is defined as follows:

$$B_s(a_1,a_2)=\int_{-\infty}^{+\infty}\psi_s(a,\tau)\psi_s(a_1,\tau)\psi_s(a_2,\tau)d\tau \tag{3}$$

Where $1/a=1/a_1+1/a_2$ is equal to $f=f_1+f_2$. The normalized bispectrum, or bicoherence, is defined as:

$$b_s(a_1,a_2)=\frac{B_s(a_1,a_2)}{\sqrt{P_s(a)P_s(a_1)P_s(a_2)}} \tag{4}$$

$$\text{With}\quad P_s(a)=\int_{-\infty}^{+\infty}\left|\psi_s(a,b)\right|^2 db \tag{5}$$

The bicoherence spectrum is used to detect the nonlinear disturbance interactions. The bicoherence measures the coherence degree amongst a wave triad due to wave coupling.

Conditional sampling and phase-average

Wavelet transform can be devoted to identify singular structure in time series signals by presetting a conditional threshold level on wavelet coefficients. If the wavelet coefficient satisfies the sampling conditions (its modulus is exceeding the threshold level), the wavelet coefficient will be preserved, which means the wavelet coefficient is in the

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incidence of the singular structure. Otherwise, the wavelet coefficient will be set as zero, which means that the wavelet coefficient is independent of the singular structure.

$$\psi_s(a,b)=\begin{cases}\psi_s(a,b) & if \quad |\psi_s(a,b)|>L \\ 0 & otherwise\end{cases} \tag{6}$$

Once the transient signatures have been segmented from the experimental output signals, the eigenfuction waveform of the universal signature signals can be obtained by superposition and average these conditional sampled signals:

$$< s(t) >= \frac{1}{N}\sum_{i=1}^{N} s(b_i + t) \quad t \in [-a, a] \tag{7}$$

RESULTS AND DISCUSSION

Figure 1 shows the multi-scale wavelet cross-spectrum providing the developing details of the frequency interaction of disturbance in hypersonic boundary layer. The interaction leads to new disturbance modulation arising and the band range of disturbance frequencies becomes broader with the boundary layer developing downstream. The disturbance develops across a broader frequency range and two significant frequency peaks appear gradually representing the first mode and the second mode respectively.

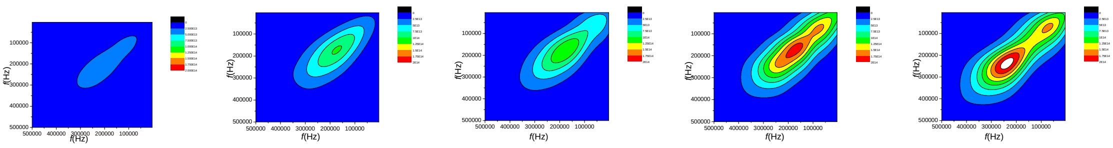

Fig.1 wavelet cross-spectrum of the temperature signals at different stations along one generatrix on the cone
(a) X=195mm, (b) X=285mm, (c) X=330mm, (d) X=375mm, (e) X=480mm

The power spectrum calculated for the extracted unstable disturbance signals presented in figure 2 provides developing details of the unstable disturbance in spectrum space. With the development of the unstable disturbance, the second mode becomes dominant with its amplitude growing quickly.

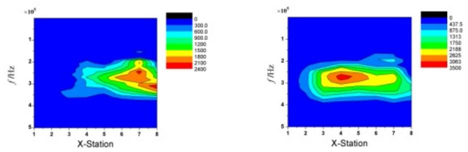

Fig.2 Power spectrum of the unstable disturbance at 8 stations along one generatrix on the cone surface
1-X=165mm,2-X=195mm,3-X=240mm,4-X=285mm,5-X=330mm,6-X=375mm, 7-X=420mm, 8-X=480mm

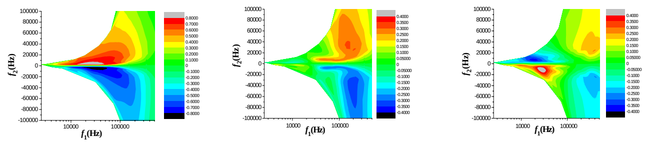

Fig.3 bicoherence spectrum at different stations along one generatrix on the cone
(a)X=165mm (b) X=420mm, (c) X=480mm

CONCLUSIONS

The study results show that wavelet analysis is a powerful tool in studying hypersonic laminar-turbulent transition. The first mode instability gives rise to frequency shifts to higher unstable modes at the early stage of hypersonic laminar-turbulent transition. The second mode instability governs the last stage of instability and final breakdown to turbulence with multi-scale disturbances growth. Nonlinear interactions of the second mode disturbance are the final dominant instabilities of laminar-turbulent transition for hypersonic shear flows.

References

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