TOWARD A SIMPLIFIED WALL-MODELED LARGE EDDY SIMULATION AT HIGH REYNOLDS NUMBER

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Summary The purpose of present study is to propose a simplified wall-modeled large eddy simulation technique by providing mean wall shear stress as a wall boundary condition without resolving near-wall region. As test flow problems, we consider two canonical flows at high Reynolds number: turbulent channel and boundary layer flows. The present technique provides good predictions of the mean statistics for both flows, although the first off-wall grid locates far away from the wall, $y^+ = O(10^2 \sim 10^3)$.

INTRODUCTION

Computing wall-bounded turbulent flow at high Reynolds number using direct numerical simulation (DNS) or large eddy simulation (LES) is prohibitively expensive due to their severe grid-resolution requirement in the wall region [1,2]. Thus, the importance of wall modeling in LES at high Reynolds number has been emphasized [1,2] and various techniques of wall-modeled large eddy simulation (WMLES) have been suggested so far [3]. One of the WMLES techniques is to provide an instantaneous wall shear stress, instead of no-slip condition, as a wall boundary condition without resolving near-wall turbulence [4-7]. Although they showed fairly good predictions of the logarithmic profile in turbulent channel flow, these approaches contain some arbitrariness in determining parameters and/or require non-negligible computational overheads due to the iterative process for the optimization, which has hampered their application to complex turbulent flows at high Reynolds number. In the present study, we examine a possibility of using the mean (rather than instantaneous) wall shear stress as a boundary condition for WMLES of wall-bounded turbulent flow at high Reynolds number. As a first step, we consider two canonical flows, turbulent channel and boundary layer flows, and apply the boundary condition without resolving turbulent flow near the wall.

NUMERICAL DETAILS

The wall boundary condition for WMLES is given as

$$\mu \frac{\partial u}{\partial y}\bigg|_w = \mu \overline{\frac{\partial u}{\partial y}}\bigg|_w, \; v_w = w_w = 0, \tag{1}$$

where *u*, *v*, and *w* denote the streamwise, wall-normal, and spanwise velocity components, respectively, *y* is the wall-normal direction, μ is the viscosity, and an overbar denotes the mean value. In the case of channel flow, the wall shear stress is directly connected to the mean pressure gradient to drive a flow inside a channel: $\mu \partial \bar{u} / \partial y|_w = -\delta \, dP / dx$, where δ is the channel half height and $-dP / dx$ is the mean pressure gradient. The periodic boundary condition is given in the streamwise (*x*) and spanwise (*z*) directions. For turbulent boundary layer flow, an empirical correlation of the wall shear stress with the Reynolds number is known, $C_f = 0.024\,\mathrm{Re}_\theta^{-1/4}$, and used here to provide the mean wall shear stress, where C_f is the skin friction coefficient and Re_θ is the Reynolds number based on the free stream velocity U_∞ and momentum thickness θ. The boundary condition on the top of domain is $u=U_\infty$, $\partial u / \partial y = \partial w / \partial y = 0$, the periodic boundary condition is imposed in the spanwise direction, and a convective boundary condition is given at the exit. A modified rescaling method based on the method of Lund *et al*. [8] is used to generate turbulent inlet boundary condition. The filtered incompressible Navier-Stokes equations are solved using a semi-implicit fractional step method and second-order central difference scheme is used in space. For the semi-implicit treatment, the Crank-Nicolson method and a third-order Runge-Kutta method are applied to the diffusion and convection terms, respectively.

RESULTS AND CONCLUSIONS

Table 1 shows the computational parameters used in this study. The grids are distributed uniformly in all three directions. Note that the first off-wall grid locates at $y^+ = O(10^2 \sim 10^3)$ depending on the Reynolds number. As a subgrid-scale (SGS) model for LES, a dynamic global model [9,10] is used. Before conducting WMLES, LES of turbulent channel and boundary layer flows (with no-slip condition) is performed at $\mathrm{Re}_\tau = 4000$ and $\mathrm{Re}_\theta = 1410$, respectively, with the grids shown in Table 1. As shown in Fig. 1, the velocity profiles are very different from the log law, indicating that LES without resolving near-wall region with no-slip condition causes serious errors.

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Table 1 Domain size and grid resolution (θ_0 denotes the inlet momentum thickness and + denotes the wall unit).

Flow	Re	Domain size			Number of grid points			Resolution		
		L_x	L_y	L_z	N_x	N_y	N_z	Δx^+	Δy^+	Δz^+
Channel	$Re_\tau = 4000$	$2\pi\delta$	2δ	$2\pi\delta / 3$	32	33	32	785	250	262
	$Re_\tau = 20000$	$2\pi\delta$	2δ	$2\pi\delta / 3$	32	33	32	3927	1250	1309
Boundary layer	$Re_\theta = 1410\text{-}3550$	$600\theta_0$	$40\theta_0$	$50\theta_0$	193	33	33	195	78	98

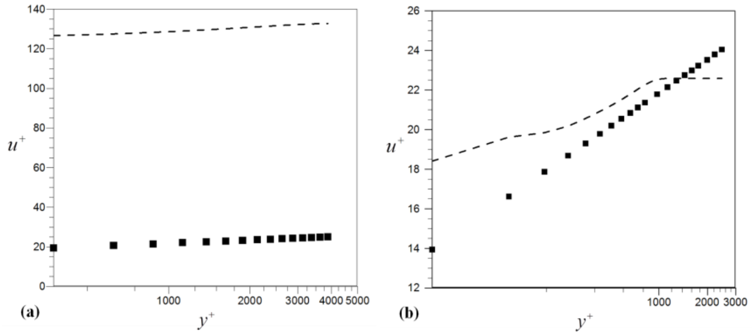

Fig. 1 Mean streamwise velocity from LES with no-slip boundary condition: (a) channel (Re_τ = 4000); (b) boundary layer (Re_θ = 1410). – –, LES; ■, $u^+ = 2.44 \log y^+ + 5$.

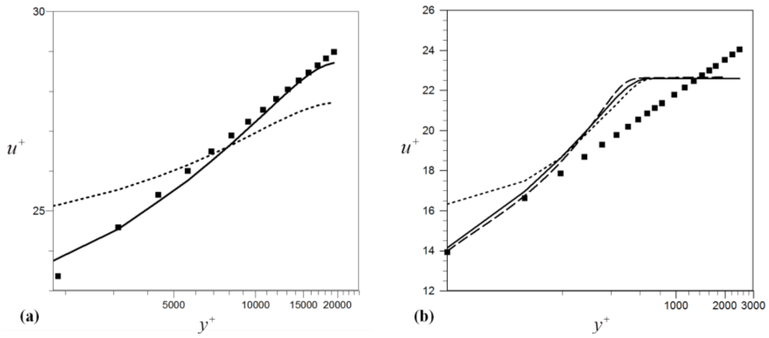

Fig. 2 Mean streamwise velocity from WMLES: (a) channel (Re_τ = 20000); (b) boundary layer (Re_θ = 1410). ——, Mean wall shear stress boundary condition; - - -, mean wall shear stress boundary condition without SGS model; — —, DNS [11]; ■, $u^+ = 2.44 \log y^+ + 5$.

Fig. 2 shows the mean streamwise velocities in wall units obtained from mean wall shear stress boundary condition with and without SGS model, respectively. As shown, the mean wall shear stress boundary condition without SGS model (i.e. coarse DNS) fails to predict reasonable results for both cases. This indicates that a SGS model also plays an important role in WMLES. On the other hand, the present velocity profiles are in good agreements with the log-law or previous DNS data, although the first off-wall grid point is far away from the wall. This implies that the logarithmic and wake profiles in wall bounded flow can be successfully predicted without resolving near-wall region, when an accurate mean wall shear stress is applied at the wall. For general wall-bounded flow, the mean wall shear stress distribution is *a priori* unknown. Currently, a dynamic approach is taken to obtain accurate mean wall shear stress during the computation, whose results will be given in the final presentation.

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