GEOMETRICAL FEATURES OF STREAMLINES AND STREAMLINE SEGMENTS IN TURBULENT FLOWS

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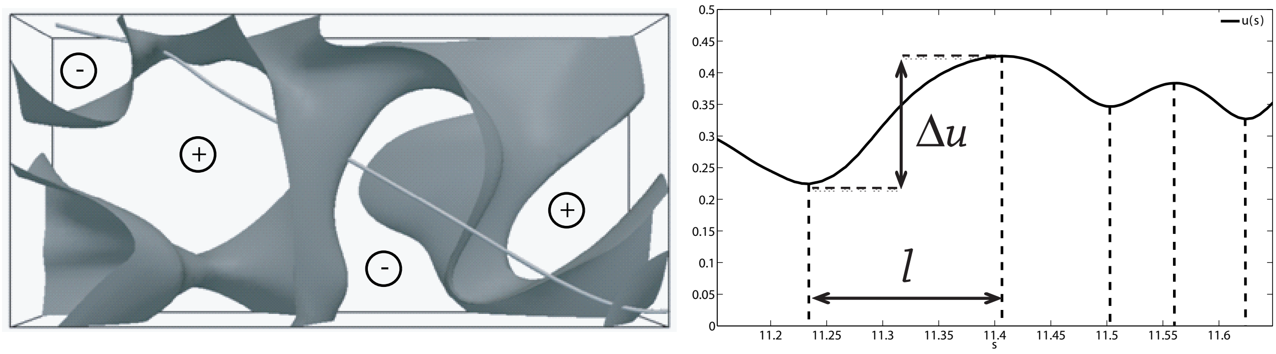
Summary Streamlines are partitioned into segments following [1]. It is shown that the ending points of streamline segments define an isosurface in space in which also all local extrema of the turbulent kinetic energy lie. As stagnation points form a sub-group of local minima of the turbulent kinetic energy field, these too are part of the isosurface. Two different measures for the curvature of streamlines are derived, which are connected by the divergence of the directional tensor. The probability density functions (pdfs) of the different measures for the curvature are analyzed and it is shown that the scaling of these is to a large extend dominated by regions in the flow close to the isosurface. While the behavior for small values of the curvatures is different for the different measures, it is dominated by regions around stagnation points for large values of the curvature and obeys the same scaling laws.

INTRODUCTION

Streamlines have recently received attention as natural geometries of turbulent flows [1, 2]. The idea of the latter work is to extend to theory of dissipation elements [3, 4] to the turbulent velocity field itself. As streamlines are a-priori infinitely long, [1] proposed to partition streamlines into segments based local extrema of u along the streamline, i.e. points where the gradient of u in streamline direction $u_s \equiv t_i \partial u/\partial x_i = \partial u/\partial s$ vanishes. Streamline segments can further be characterized as positive ones for $u_s > 0$ and negative ones for $u_s < 0$. It turns out that when u_s is treated as a scalar field all streamline segments ending points lie in an isosurface defined by $u_s = 0$, which divides space into two regions, one of which contains all positive segments while the other contains all negative segments. Figure 1 (a) shows this isosurface as well as a streamline entering the box at the top left corner and leaving it at the bottom right corner. Figure 1 (b) shows the variation of u along the streamline as well as the streamline segments. It follows readily that apart from the extreme points along the streamlines, all local extreme points (of the field of the absolute value of the velocity u and the turbulent kinetic energy $k = u^2/2$), i.e. points where $\nabla u = \nabla k = 0$, also lie in the isosurface. Note that stagnations points, i.e. critical points of the velocity field where locally all velocity components vanish simultaneously, form a sub-group of local minimum points. The isosurface itself can further be subdivided into two (not necessarily simply connected; isolated islands can exist) parts, one of which contains all minimum points (minimal surface), while the other contains all maximum points (maximal surface). The demarcation line between these two regions is the ensemble of points where streamlines are tangent to the isosurface. Figure 2 (a) shows such a partitioning of the surface.



(a) (b)

Figure 1. (a) Isosurface defined by $u_s = 0$ and streamline. (b) Variation of u along the streamline over the arclength and one segments.

The aim of the present work is to provide insight into the geometrical features of streamlines with respect to the isosurface. We are especially interested in the curvature of streamlines and its statistics whose scaling will be shown to be related to regions around the isosurface.

ANALYSIS

Let us define the directional tensor of streamlines

$$T_{ij} = t_i t_j. \tag{1}$$

Taking the divergence of Eq. (1), we obtain

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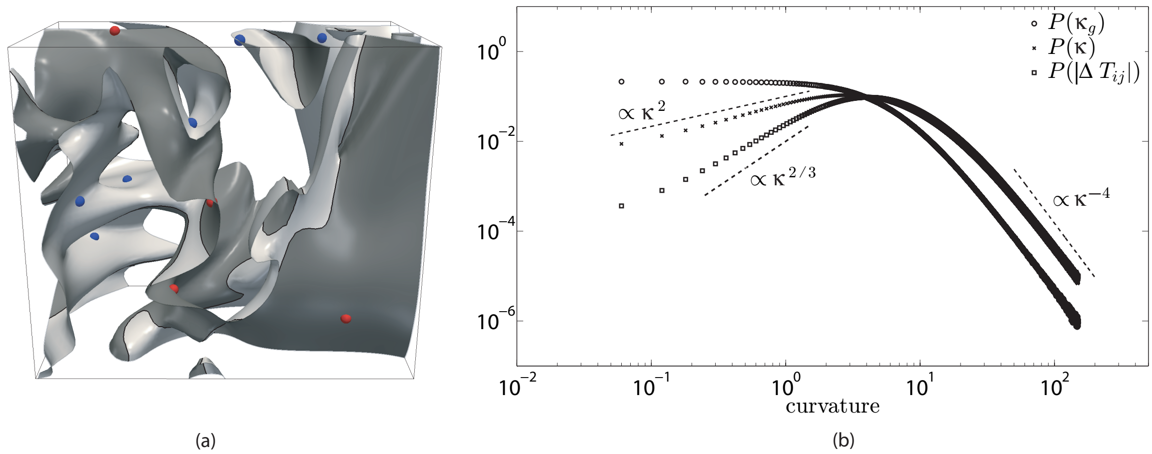

Figure 2. (a) Subdivision of the $u_s = 0$ isosurface into minimal (light grey) and maximal (dark grey) surface regions with demarcation line (black). Minimum points (blue dots) of the k-field lie in the light grey region, maximum points (red dots) in the dark grey region of the surface. (b) Pdfs of the measures for the curvature and their scaling.

$$\nabla T_{ij} = \frac{\partial T_{ij}}{\partial x_j} = \frac{\partial t_i}{\partial s} + t_i \frac{\partial t_j}{\partial x_j} = \kappa_i + t_i \kappa_g, \tag{2}$$

where we denote with $t_j \partial/\partial x_j = \partial/\partial s$ the gradient in direction of the streamline and with $\kappa_g = \partial t_j/\partial x_j$ the Gaussian curvature of streamlines. Wang [1] has shown that for incompressible flows the Gaussian curvature can be expressed as $\kappa_g = -u_s/u$, showing that it vanishes at local extrema of the absolute value of u along the streamline, i.e. in the isosurface. Taking the absolute value of Eq. (2) we obtain the following relationship

$$|\nabla T_{ij}| = \sqrt{\kappa^2 + \kappa_g^2}, \tag{3}$$

which shows that the absolute value of the divergence of the directional tensor contains rich information about the geometry of streamlines in terms of different measures of their curvature. We will study the statistics of the different contributions of Eq. (3) based on three DNS cases of isotropic (decaying and forced) turbulence at different Reynolds numbers ranging from $Re_\lambda = 50 - 200$. Figure 2 (b) shows the probability density function (pdf) $P(\kappa_g)$, $P(\kappa)$ and $P(|\nabla T_{ij}|)$ in a log-log plot for the highest case (the scaling is the same, independent of the Reynolds number which is why at this point we only show one examplary result). The three different curves start off differently for small values of the curvature, while they all show a pronounced algebraic tail for $\kappa \to \infty$ that scales as κ^{-4}. Let us first discuss the case of the Gaussian curvature κ_g. Large values of κ_g can either be assumed for large values of u_s or small values of u. It turns out that the dominating contribution for very large curvatures result from regions with very low values of u, thus regions in the vicinity of stagnation points in the flow. Although, as has been shown above, such points lie in the isosurface, where $u_s = 0$, the region in the vicinity yields large values of κ. As we can assume that for isotropic homogeneous turbulence the three fluctuating velocity compontents u_i are joint normal, for a recent discussion of the validity of this assumption see [5], it follows that the pdf of the turbulent kinetic energy k yields a χ^2-distribution with 3 degrees of freedom: $P(k) \propto k^{1/2} \exp -k/2$. From this it is straightforward to obtain the scaling of the pdf of the inverse of the absolute value of the velocity field $x = u^{-1}$ as $P(x) \propto x^{-4} \exp -x^{-2}$ which for small values of u, large values of x scales as $P(x \to \infty) \propto x^{-4}$. This scaling seems to be valid for all three different curves. For small values of κ_g it turns out that the pdf is dominated by the region around the isosurface, where $u_s \approx 0$, rather than by large values of u. As one can assume that at least for small values of u_s, its pdf is close to Gaussian it follows $P(\kappa_g \to 0) \approx const.$. The scaling of $P(\kappa) \propto \kappa^2$ for $\kappa \to 0$ can be explained in analogy to the findings in [6] who considered the curvature of particle paths in turbulent flows. Having explained the two branches of the pdfs $P(\kappa_g)$ and $P(\kappa)$, the scaling of $P(|\nabla T_{ij}|)$ follows from Eq. (3).

References

[1] L. Wang. On properties of fluid turbulence along streamlines. *J. Fluid Mech.*, 648:183–203, 2010.

[2] P. Schaefer, M. Gampert, and N. Peters. The length distribution of streamline segments in homogeneous isotropic decaying turbulence. *submitted to Phys Fluids*.

[3] L. Wang and N. Peters. The length scale distribution function of the distance between extremal points in passive scalar turbulence. *J. Fluid Mech.*, 554:457–475, 2006.

[4] L. Wang and N. Peters. Length scale distribution functions and conditional means for various fields in turbulence. *J. Fluid Mech.*, 608:113–138, 2008.

[5] M. Wilczek, A. Daitche, and Friedrich R. On the velocity distribution in homogeneous isotropic turbulence: correlations and deviations from gaussianit. *J. Fluid Mech*, 676:191 – 217, 2011.

[6] W. Braun, F. De Lillo, and B. Eckhardt. Geometry of particle paths in turbulent flows. *J. of Turbulence*, 7:1–10, 2006.