MACH NUMBER SIMILARITY OF LARGE-SCALE STRUCTURES IN COMPRESSIBLE CHANNEL FLOWS[a)]

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Summary The large-scale structure of wall-bounded compressible turbulence is studied by analyzing channel flow DNS data up to Mach number (Ma)=3. We use the concept of *velocity-vorticity correlation structure* (VVCS), defined by high values of two-point, velocity-vorticity cross-correlation tensor. VVCS displays *Ma*-similarity of its geometrical features throughout the channel. VVCS shows that the nonlocal link between VVCS at y_s and the velocity fluctuations at y_r yields a universal function under the density-weighted transformation (DWT) (semi-local coordinates proposed by Huang *et al.* [3]). Moreover, spanwise spacing (D_z) exhibits also a *Ma*-independent linear increase with y_r. Thus, the VVCS provides geometrical interpretation of the Morkovin's hypothesis.

INTRODUCTION

Low-speed streaks and quasi-streamwise vortices have been extensively studied experimentally and computationally, as the dominant coherent structures contributing to near-wall turbulent kinetic-energy production, drag and heat transfer [1]. However, their relationship to outer-region flow is still unclear. Further, quantitative study of these structures is needed for modeling of wall-bounded turbulence in engineering applications. Study of two-point streamwise velocity-correlation suggests that the spanwise spacing between streaks in the sublayer is approximately $\lambda_z^+ \sim 100$, and increases with the wall-normal distance towards the outer region [2]. On the other hand, compared to streaks, the streamwise vortices are much less studied, even far less in compressible wall-bounded flows. The density-weighted transformation (also called semi-local coordinate) proposed by Huang *et al.* [3] seems to improve the van Driest transformation using density at the wall. We study study the connection between streamwise vortices and streaks by extracting VVCS from DNS [4] and derive the scaling behavior of turbulent vortices at moderate *Ma*.

VELOCITY-VORTICITY CORRELATION STRUCTURE

The VVCS is the high correlation regions identified using two-point cross-correlation tensor of velocity and vorticity. Considering a three-dimensional turbulent field, we fix a reference point to measure velocity fluctuations and move the other 'probe' to record the vorticity fluctuations, then calculate the time-averaged two-point correlation coefficient tensor field.

$$R_{u_i\omega_j}(\Delta x, y, \Delta z; y_r) = \frac{\overline{u_i(x, y_r, z)\omega_j(x+\Delta x, y, z+\Delta x)}}{u_{i,rms}(y_r)\omega_{j,rms}(y)}. \tag{1}$$

where, $u_i(x, y_r, z)$ and $\omega_j(x+\Delta x, y, z+\Delta z)$ are instantaneous velocity at point $A = (x, y_r, z)$ and vorticity at point $B = (x+\Delta x, y, z+\Delta z)$, respectively. The separation of the two points is $(\Delta x, y-y_r, \Delta z)$. Then, $R_{u_i\omega_j}$ establishes a three-dimension correlation coefficient field around y_r. At a chosen threshold α, i.e. $|R_{u_i\omega_j}| \geq \alpha$, we obtain the most correlated vorticity volume to the velocity fluctuation at the reference point y_r. Typically, $\alpha \approx 0.1$, defines a clear geometry for each of the nine components. Hence, the VVCS, defined by the iso-surface $|R_{u_i\omega_j}| = \alpha$, is a good tool to reveal statistical structures of an inhomogeneous turbulence field, as shown by the analysis of compressible turbulent channel flows [4]. Note that $R_{u_i\omega_j}$ is a 3-D function dependent on y_r. The analysis has been carried out to DNS data of compressible channel flow up to $Ma = 3.0$ with 4×10^6 grid points and with Reynolds number *Re* of 3000 (based on the centerline velocity and the channel half-width). Focusing on a particular component of VVCS, namely $VVCS_{u\omega_x}$, we find two kinds of structures: (a) the *first* consists of two opposite-signed quasi-streamwise structures almost parallel to the wall, termed as streamwise correlation structure (*SCS*); (b) the *second* displays two flat substructures with opposite signs attached to the wall having a long streamwise extent, called near-wall structure (*NCS*). They seem to display a three-dimensional geometry consistent with streaks and counter-rotating streamwise vortices (see Figure 1(a)). In addition, the NCS is always located under the SCS. As y_r increases, the SCS changes from the inclined streamwise structure for $y_r^+ < 140$ (Figure 1(a)) to the blob-like structure for $y_r^+ > 140$ (Figure 1(b)), whereas the geometry of NCS remains unchanged at all y_r [4].

THE SCALING BEHAVIOR AND *MA*-INDEPENDENCE OF SCS AND NCS

Once a VVCS is computed from a turbulent field at a given reference point y_r, we define y_s the location of the vorticity component with the maximum correlation. The relation $y_s(y_r)$ represents a non-local link between the reference velocity fluctuations and the associated VVCS. Figure 2(a) shows the quantitative relations betwwen y_s and y_r (in wall units) and

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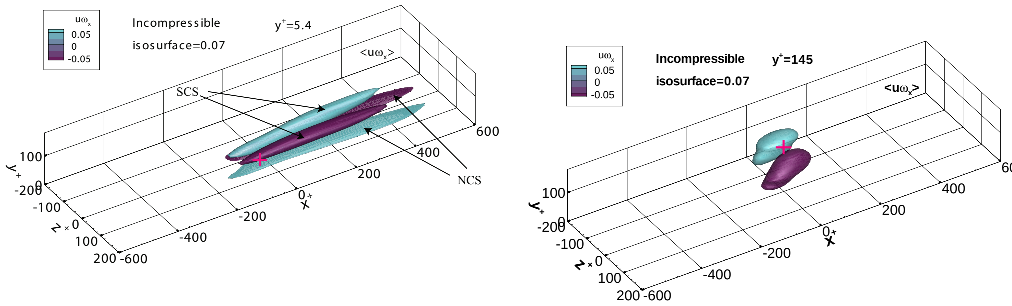
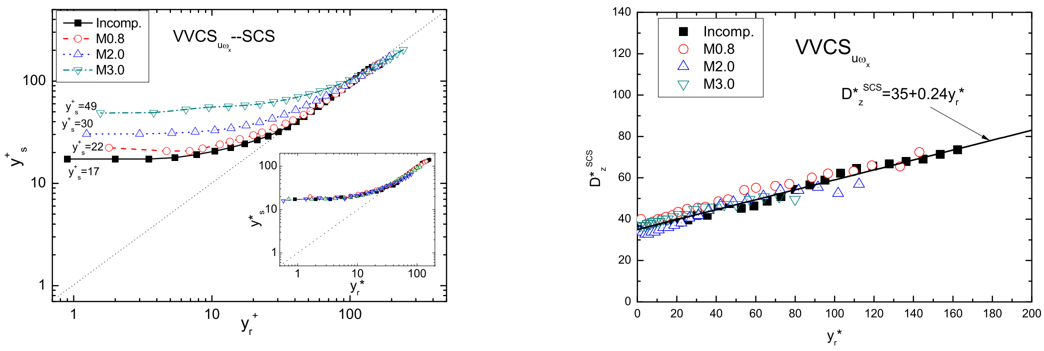
that by applying DWT. For both incompressible and compressible channel flows, y_s^+ is nearly constant for $y_r^+ < 10$ in the viscous sublayer, indicating that a critical (streamwise) vorticiy layer is mostly responsible for the velocity fluctuations very close to wall. In addition, the location of this critical layer for SCS monotonically increases with *Ma*, indicating that the thickness of the viscous sublayer increases (in wall units). The profile $y_s^+(y_r^+)$ displays remarkable similarity at different *Ma* under the DWT [3] (see the inset of Figure 2(a)). The *Ma*-independence supports the Markovin hypothesis that the statistical correlation between the velocity and vorticity is entirely determined by the mean density. An empirical fitting of the *Ma*-independent function can be expressed as: $y_s^* = f(y_r^*) = \exp\left[2.82\left(1 + 0.56(y_r^*/200) - (y_r^*/200)^2\right)\right]$.



(a) $y_r^+ = 5.4$ (b) $y_r^+ = 145$

Figure 1. Geometry of $VVCS_{u\omega_x}$.



(a) y_s vs. y_r (b) D_z vs. y_r

Figure 2. (a) The structure wall-normal distance for SCS $y_s^+(y_r^+)$ at four Mach numbers. The inset is $y_s^*(y_r^*)$ under the density-weighted transformation. (b) The spanwise spacing of SCS, $D_z^* = 2|z_s^*|$ in semi-local coordinates.

Furthermore, the spacing between the two parallel regions of the SCS, D_z^*, is found to be linear in y_r^*: $D_{z,SCS}^* \approx 0.24y_r^* + 35$ (Figure 2(b)). The minimum spanwise spacing of SCS occurs at $y_r = 0$ is 35, slightly smaller than Blackwelder's estimation of 50 [5]. This *Ma*-independent spacing of SCS also demonstrates that the Morkovin's hypothesis reveals an underlying geometrical similarity in compressible channel flow. The nature and the extension of this similarity to other wall-bounded flows with different boundary conditions need further study.

CONCLUSIONS

The VVCS seems to provide a useful tool to study quantitatively the statistical geometrical structures throughout a compressible channel flow, not only for various distance from the wall, but also for *Ma*-effects. Two pairs of streamwise VVCS of u and ω_x, corresponding to long steaks and quasi-streamwise vortices, respectively, are examined and are shown to change its geometry with y_r. In particular, a quantitative relation between the location of the reference velocity and that of the most correlated vorticity is suggested to characterize the geometrical property of the underlying turbulent field. Another quantitative measure, spanwise spacing D_z of SCS, is shown to display a linear function of y_r^* for compressible channel flow. Both measures collapse under the DWT [3]. The results yield a geometrical context of the Morkovin's hypothesis.

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