SELECTIVE REDUCTION OF TRIAD INTERACTION IN HALL MAGNETOHYDRODYNAMIC TURBULENCE

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Summary A detailed analysis of energy transfer processes due to triad mode interactions in freely decaying, homogeneous, isotropic Hall magnetohydrodynamic (HMHD) turbulence is performed through Fourier analyses. We analyzed six snapshot datasets that were taken from such a period to allow the turbulence to develop sufficiently with a nearly constant magnetic Reynolds number. Because the Fourier energy spectra in these snapshots show remarkable agreement after the normalization in terms of the dissipation rates and the diffusion coefficients, they are considered as a universal equilibrium state. We analyzed spherical shell averaged triad mode interactions of each term of HMHD equations in detail and found that normalized mode interaction amplitudes due to the fluid advection and Hall terms effects are selectively reduced, while those due to the other two terms, i.e. the induction and Lorentz force terms are retained rather stationarily.

INTRODUCTION

The HMHD system is regarded as one of the simple models to include the two-fluid effect in magnetized plasma dynamics (e.g.[1]). Energy transfer process of HMHD turbulence is investigated in detail by Mininni et al.[2]. They found some very interesting interaction features, e.g. backscatter of magnetic energy due to the Hall term effect, though their analysis did not go into the detailed analysis of triad mode interaction that is found in, for example, Ref.[3]. Recently authors confirmed by the Fourier and wavelet analyses that the backscatter process is intrinsic to the Hall term effect and the mode interaction features are quite different between small and large scales [4].

BASIC EQUATIONS AND NUMERICAL ANALYSIS METHODS

Basic equations for an incompressible Hall magnetohydrodynamic system in dimensionless form are given by the following equations:

$$\nabla\cdot\boldsymbol{u} = \nabla\cdot\boldsymbol{b} = 0, \tag{1}$$
$$\partial_t\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla P + \boldsymbol{j}\times\boldsymbol{b} + \nu\triangle\boldsymbol{u}, \tag{2}$$
$$\partial_t\boldsymbol{b} = \nabla\times(\boldsymbol{u}\times\boldsymbol{b}) - \epsilon\nabla\times(\boldsymbol{j}\times\boldsymbol{b}) + \eta\triangle\boldsymbol{b}, \tag{3}$$

where $\boldsymbol{u}$, $\boldsymbol{b}$, P, $\boldsymbol{j} := \nabla\times\boldsymbol{b}$, ν, ϵ, η are velocity field, magnetic field, generarized pressure, current, kinematic viscosity, the Hall parameter, and resistivity, respectively. We used the same direct numerical simulation (DNS) that was reported in Ref.[5], in which the parameters are set to $\nu = \eta = 0.001$ and $\epsilon = 0.05$. The DNS is carried out by using the pseudospectral methods with 512^3 grid points on the box and the 4-th order Runge-Kutta-Gill method. We analyzed here six snapshot datasets at the time $t = 1.0$, 1.5, 2.0, 2.5, 3.0, and 3.5.
In order to compare quantitatively the different time data of the freely decaying turbulence, each field data is decomposed accoding to its Kolmogorov scale l_η. Furthermore to identify the contribution of various spatial scale in the unit of the power of some appropriate scale ratio (we take $\sqrt{2}$ here), the velocity and magnetic fields are decomposed as follows

$$\boldsymbol{f}(\vec{x},t) = \sum_j \boldsymbol{f}_j(\vec{x},t), \quad \text{where} \quad \boldsymbol{f}_j(\vec{x},t) = \sum_{k_\eta(t)/(\sqrt{2})^{j+1}<|\vec{k}|\le k_\eta(t)/(\sqrt{2})^j} \hat{\boldsymbol{f}}(\vec{k},t)\exp\left(i\vec{k}\cdot\vec{x}\right), \tag{4}$$

where $\boldsymbol{f}$ stands for $\boldsymbol{u}$ or $\boldsymbol{b}$, and $k_\eta(t)$ is the Kolmogorov wave number of the field of the snapshot data time. That is, the Fourier space is divided into the spherical shell bands and the characteristic oscillation wave length (say λ_j) and the envelope size of fluctuation of $\boldsymbol{f}_j$ (in other words, the window width of the band pass filter Δ_j) obey the scaling relations given by $\lambda_j \propto \sqrt{2}^j l_\eta,\ \Delta_j \propto \sqrt{2}^j l_\eta$.
Taking inner product of Eqs.(2) and (3) with $\boldsymbol{u}_l$ and $\boldsymbol{b}_l$, respectively, one obtains the shell averaged energy budget equation of HMHD system as follows:

$$\partial_t E_U(l) = \sum_{j,k} T_{UU}(j,k,l) + \sum_{j,k} T_{UB}(j,k,l) - D_U(l),$$
$$\partial_t E_B(l) = \sum_{j,k} T_{BU}(j,k,l) + \sum_{j,k} T_{BB}(j,k,l) - D_B(l)$$

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where $E_U(l) = \int |\boldsymbol{u}_l|^2 \mathrm{d}^3\vec{x}$, $E_B(l) = \int |\boldsymbol{u}_l|^2 \mathrm{d}^3\vec{x}$, $D_U(l) = \nu \int |\nabla \times \boldsymbol{u}_l|^2 \mathrm{d}^3\vec{x}$ and $D_B(l) = \eta \int |\nabla \times \boldsymbol{b}_l|^2 \mathrm{d}^3\vec{x}$, and the shell averaged triad mode interactions

$$T_{UU}(j,k,l) = -\int ((\boldsymbol{u}_k \cdot \nabla)\boldsymbol{u}_j) \cdot \boldsymbol{u}_l \mathrm{d}^3\vec{x}, \qquad T_{UB}(j,k,l) = \int (\boldsymbol{j}_j \times \boldsymbol{b}_k) \cdot \boldsymbol{u}_l \mathrm{d}^3\vec{x},$$

$$T_{BU}(j,k,l) = \int (\nabla \times (\boldsymbol{u}_j \times \boldsymbol{b}_k)) \cdot \boldsymbol{b}_l \mathrm{d}^3\vec{x}, \qquad T_{BB}(j,k,l) = -\epsilon \int (\nabla \times (\boldsymbol{j}_j \times \boldsymbol{b}_k)) \cdot \boldsymbol{b}_l \mathrm{d}^3\vec{x}.$$

These forms are carefully chosen to retain the covariance under the arbitrary change of coordinate system (cf.[6]), though in this paper we treat only the Cartesian coordinate system case with periodic boundary condition. These integrals satisfiy the following reciprocal relations: $T_{UU}(j,k,l) = -T_{UU}(l,k,j)$, $T_{UB}(j,k,l) = -T_{BU}(l,k,j)$, $T_{BB}(j,k,l) = -T_{BB}(l,k,j)$.

NUMERICAL RESULT AND DISCUSSION

In Figure 1 the time development of the typical profiles of triad mode interactions are shown. The contours are drawn according to the moduli of each T_{XY}'s and the increments of them are fixed to $0.05\epsilon^{(b)}(t)$ for convenience of intuitive comparison between different terms and times.

It is very remarkable that the amplitudes of normalized mode interactions due to the fluid advection and the Hall term effects are selectively reduced, while those of the indunction and Lorentz force terms retained in some rather stationary manner.

Depression of nonlinearity in fully developed turbulence attracted interests of turbulent researchers and is investigated for the neutral fluid case (e.g.[7, 8]) and the magnetohydrodynamic (MHD) fluid case [9]. Their analysis were mainly based on the comparison of turbulent fields with the phase randomized fields. In the present study, we proposed the direct comparison of the amplitude of triad interactions after the normalization by the disspation rate and found that selective depression of triad mode interaction in HMHD turbulence. Investigation of the physical reasons of this novel selective reduction phenomenon is now underway.

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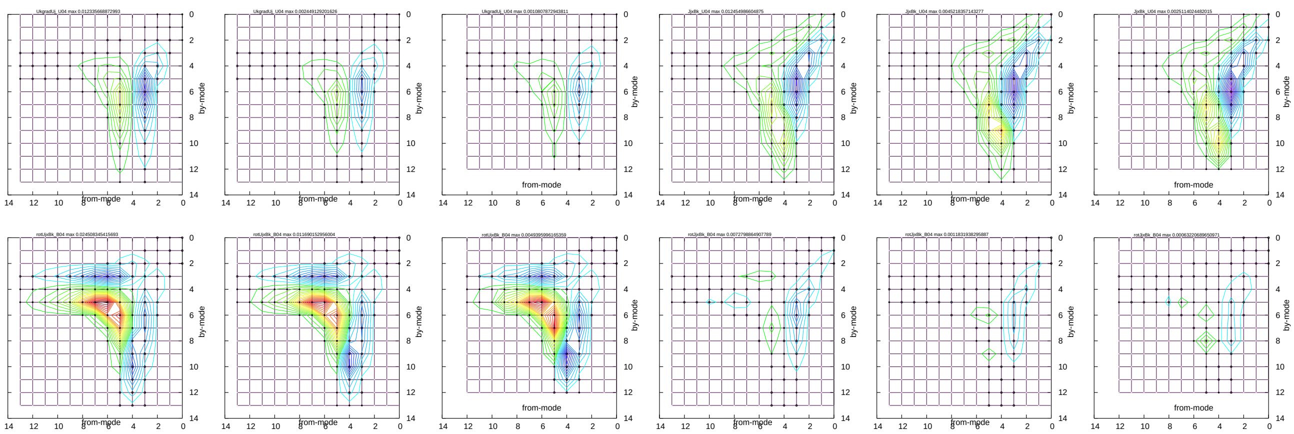

Figure 1: time evolution of the typical profiles of shell averaged triad mode interactions $T_{XX}(j,k,4;t)$. Each set of three panels at top left: T_{UU}, top right: T_{UB}, bottom left: T_{BU}, and bottom right: T_{BB}. Three panels in each set correspond to the dataset at $t = 1.5$, 2.5 and 3.5 (left to right) in dimensionless time. Abscissa: number of j, ordinate: number of k; contours in green to red: energy receiving interactions, blue to purple: energy losing ones.