A MULTI-LAYER DESCRIPTION OF SUBGRID-SCALE EFFECTS

Ning Hu[*,**], Huan Liu[**] & Zhen-Su She[** a)]
[*] *China Academy of Aerospace Aerodynamics, Beijing, 100074, China*
[**]*State Key Laboratory for Turbulence and Complex Systems, Peking University, Beijing, 100871, China*

Summary Large eddy simulations (LES) of wall-bounded turbulence suffer from the high anisotropy of near-wall coherent structures. The subgrid-scale (SGS) effects have complex dependences with grid sizes and wall distance. In this paper the inter-relation between coarse-grid and SGS fluctuation fields are investigated, using a ratio r of SGS stress to the total Reynolds stress – also called the Speziale function. A multi-layer model is introduced to describe r, with a length scale function ℓ_W which describes the variation with wall distance. The obtained r model agrees well with the direct numerical simulation data at all distance from the wall and all grid sizes.

INTRODUCTION

In practical applications with LES of wall-bounded turbulent flows, a common situation is that the grid is very coarse or the Reynolds number is very high. In such a case much important dynamics information is contained in the SGS motion, which has to be properly modeled. To this end, many hybrid RANS/LES methods have been proposed. However, the relationship among grid size, wall distance and the sizes of coherent structures has not been well understood yet, and many models depend on artificial tuning of parameters or other information input.

Recently, She et al.[1] has proposed a new approach for studying the non-homogeneous turbulence from the statistical view, called the Structural Ensemble Dynamics (SED) approach. This approach has been successfully applied in the study of RANS model for wall-bounded turbulence[2,3]. In this paper, we further use this approach to study the grid-size effects of SGS stress, to provide information for SGS model constructing.

SGS EFFECTS AND MULTI-LAYER MODEL

LES equation is a filtered Navier-Stokes equation. The equation for the filtered momentum reads:

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{1}{Re}\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \tag{1}$$

where τ_{ij} is the subgrid-scale (SGS) stress. Ensemble-average the above equation yields:

$$\frac{\partial W_{ij}}{\partial x_j} = -\frac{\partial P}{\partial x_i} - \frac{\partial T_{ij}}{\partial x_j} + \frac{1}{Re}\frac{\partial^2 U_i}{\partial x_j \partial x_j} \tag{2}$$

where $W_{ij} = \langle \bar{u}_i \bar{u}_j \rangle$ is the resolved part of total Reynolds stress and T_{ij} is the mean SGS stress. Speziale[4] introduced the function $r = T / (W + T)$ to describe the relative importance of SGS motion. In the channel flow $r = r\,(y^+, \Delta_x, \Delta_z, Re_\tau)$. Introducing a length scale $\ell_W = \ell_W(y, Re_\tau)$, we assume that $r = r(\Delta_z / \ell_W, \Delta_x, Re_\tau)$. Here ℓ_W could be viewed as the scale of Reynolds-stress-producing structures. Following the philosophy in [1], ℓ_W should have a multi-layer structure:

$$\ell_W = c_W \left(1 + \left(\frac{y^+}{y_{sub}}\right)^{2.5}\right)^{(n_1/2.5)} \left(1 + \left(\frac{y^+}{y_{buf}}\right)^{2.5}\right)^{(n_2/2.5)} \left(1 + \left(\frac{y}{1 - r_{core}}\right)^{2.5}\right)^{(n_3/2.5)} \tag{3}$$

with the thicknesses of the sub-layer $y_{sub} \approx 9.8$, the buffer-layer $y_{buf} \approx 41.5$ and the core $r_{core} \approx 0.27$ invariant with Re, and scaling behaviour $\ell_W \sim y^{n_i}$ between the layers. n_i (i = 1,2,3) are the corresponding scaling exponents.

When the grid size greatly exceeds that of the local Reynolds-stress-producing eddies, the simulation is resemble to a RANS simulation. On the other hand, for sufficiently small grid size there is good self-similarity among the scales, and the grid is appropriate for LES. Thus we assume a scaling law $r \sim (\Delta_z / \ell_W)^{n_r}$ in the variation of r with Δ_z / ℓ_W when the grid size is much smaller than the RANS limit. Denoting this limit by Δ_{RANS}, the model for r can be written as:

$$r(\Delta_z^+ / \ell_W) = \left(1 + \left(\frac{\Delta_z^+ / \ell_W}{\Delta_{RANS}}\right)^{-p_r}\right)^{(n_r/-p_r)} \tag{4}$$

where Δ_{RANS}, n_r, and the transition sharpness p_r are unknown parameters to be determined.

VERIFICATION WITH DNS DATA

In this section we use DNS data to verify the model for r. The DNS of incompressible channel flow is carried out by the spectral method at wall friction Reynolds number Re_τ=395, with the periodical domain $L_x \times L_z = 2\pi \times \pi$, and the grid number $N_x \times N_y \times N_z = 160 \times 193 \times 160$. With a top-hat filter the CG field and the SGS quantities are calculated:

---

a) Corresponding author. Email: she@pku.edu.cn.
This work is supported by National Nature Science Fund 90716008 and by MOST 973 project 2009CB724100.

Varying the filter size, the variation of Speziale function r with the filter size can be obtained. Given a choice of n_i in Eq (3), the value of r on every Δ_z/ℓ_W point is scattered for different y^+. Since we expect that the profile of $r(\Delta_z/\ell_W)$ should be independent of y^+, minimizing the error yields $n_1\approx0.5$, $n_2\approx0.225$ and $n_3\approx-0.9$.

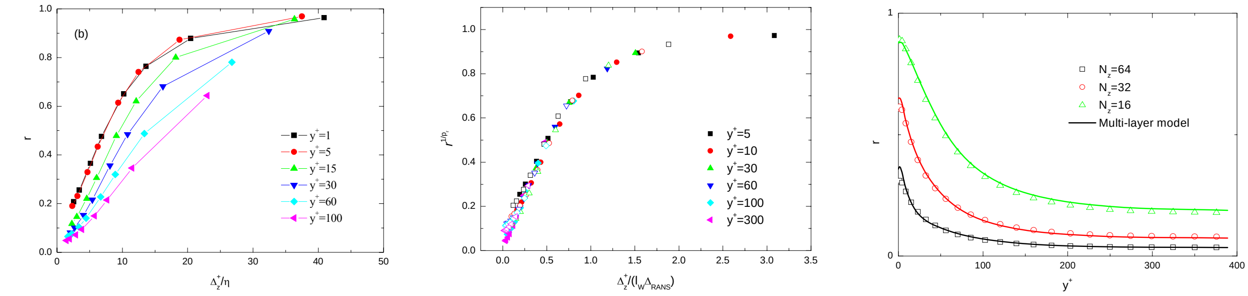

Figure 1: r varying with Δ_z normalized by (a) Kolmogorov length scale η and (b) by ℓ_W. (c) comparison of our model with DNS data.

Fig 1 shows the variation of r with normalized spanwise grid size. As the grid size increases, the SGS stress approaches to the total Reynolds stress, thus r increases with the grid size and decreases as y moves away from the wall. Variation of r with Δ_z normalized by lw is nearly independent of wall-distance y, indicating a good description of Reynolds-stress-producing eddies size. On the contrary, variation of r with Δ_z normalized by the Kolmogorov scale (suggested by Speziale [5]) cannot collapse. It is also found that the variations of Δ_{RANS} and n_r with Δ_x follow an exponential law

$$\Delta_{RANS}=89-50\exp\left[-\frac{\Delta_x}{0.4}\right],\qquad n_r=0.35+1.14\exp\left(-\frac{\Delta_x}{0.28}\right) \tag{18}$$

APPLICATION IN VERY LARGE EDDY SIMULATIONS (VLES)

Here we apply our multi-layer model in the VLES of turbulent channel flow. At each time step, calculate the spatial averaged Reynolds stress, the mean shear rate and the instantaneous mixing length:

$$\bar{U}=\frac{1}{L_x\times L_z}\int_0^{L_z}\int_0^{L_x}\bar{u}dxdz,\qquad \bar{W}=\frac{1}{L_x\times L_z}\int_0^{L_z}\int_0^{L_x}(\bar{u}-\bar{U})dxdz,\qquad \bar{\ell}_{M,GS}=\frac{\sqrt{\bar{W}}}{\bar{S}},\ \text{where } \bar{S}=\frac{d\bar{U}}{dy} \tag{15}$$

When $\bar{\ell}_{M,GS}$ differs from the theoretical profile $\ell_{M,theory}$ in [3], the ratio β between the two is used to renormalize the turbulent fluctuation: $u^{(1)}=\bar{U}+[(1-r)\beta]^{1/2}(\bar{u}-\bar{U}),\ v^{(1)}=[(1-r)\beta]^{1/2}\bar{v}$.

4 Reynolds numbers: Re_τ=395, 950, 2000 and 4000 are simulated using the same number of grids, i.e., 32×129×32. Fig 2 shows that the mean velocity profile and turbulent kinetic energy (TKE) obtained using the multi-layer model give perfect agreement with DNS or experiments for all cases, better than the corresponding results in references.

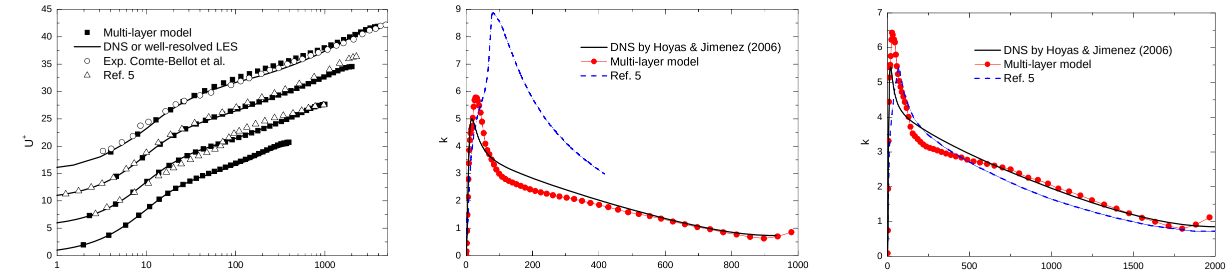

Figure 2: mean velocity profile and TKE compared with the experiment, DNS result and VLES results in [5].

CONCLUSIONS

Closure strategy of the ensemble-averaged LES equations is investigated. A multi-layer model for the Speziale function r is constructed and shown valid for all wall distance. Application to VLES calculation yields good agreement with DNS. The results demonstrate that the inter-scale properties of inhomogeneous turbulence obey the universal multi-layer similarity.

References

[1] She Z.-S.,Wu Y., Chen X., Hussain F. Understanding wall turbulence, Part I: a new closure approach. *J. Fluid Mech.*,2011; submitted.
[2] She Z.-S., Hu N., Wu Y. SED based closure model for wall bounded turbulent flow. *Acta Mech. Sin.* 2009, 25:731–736
[3] She Z.-S., Chen X., Wu Y., Hussain F. New perspective in statistical modeling of wall-bounded turbulence. *Acta Mech Sin.*, 2010
[4] Speziale C. G. Turbulence Modeling for Time-Dependent RANS and VLES: A Review. *AIAA Journal.* 1998, 36:173–184
[5] Davidson L., Peng S.H. *Int. J. Numer. Meth. Fluids*, 2003; 43: 1003-1018; Davidson L., Dahlstrom S. *Int. J. Comput. Fluid Dyn.* 2005, 19(6):415–427