ENERGY DISSIPATION IN FULLY DEVELOPED TURBULENCE

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Summary A multiscale description of the energy dissipation rate $\overline{\epsilon}$ in fully developed turbulence is investigated experimentally by the meaning of the Hilbert-Huang transform. A universal power law behavior of the energy dissipation density $\mathcal{A}(\omega)$ is obtained experimentally with an exponent $\alpha \simeq 0.37$. The energy dissipation contained in the inertial range is above 10% of the total energy dissipation, and thus cannot be neglected. Interaction between different scales is also observed, indicating a nonlocal energy cascade.

INTRODUCTION

The physical picture behind the celebrated Kolmogorov 1941 turbulent theory (K41) is Rischardson's energy cascade [1]. It is a multiscale description of turbulent flows, in which, at least in the so-called inertial range, it possesses a continuous range of scale eddies. For the high Reynolds number turbulent flows, the K41 theory predicts the famous $-5/3$ law of the Fourier power spectrum of velocity, i.e., $E_u(f) \sim f^{-5/3}$ [1]. The corresponding dissipation spectrum is [1]

$$E_\epsilon(f) = f^2 E_u(f) \sim f^\alpha, \quad \overline{\epsilon} = \int_0^{+\infty} E_\epsilon(f) df \tag{1}$$

in which $\alpha = 1/3$ for the nonintermittent Kolmogorov value. Due to the intermittent correlation, the experimental value is expected as $\alpha = 1/3 - \mu$, in which μ is the so-called intermittent parameter [1]. Note that the above relation holds only for a linear process. For a single point measurement, the energy dissipation rate is estimated as

$$\overline{\epsilon} \equiv 15\nu \frac{1}{U^2} \langle \left(\frac{\partial u}{\partial t} \right)^2 \rangle_t \tag{2}$$

in which U is mean velocity, and Taylor's frozen hypothesis has been applied. $\overline{\epsilon}$ should contains the dissipation from all eddies.

MULTISCALE DECOMPOSITION OF ENERGY DISSIPATION RATE

The method we implemented in this study in the so-called Hilbert-Huang transform [2, 3]. It is fully data-driven method, and is very suitable for nonlinear and nonstationary processes [2]. It consists of two steps, i.e., Empirical Mode Decomposition and Hilbert Spectral Analysis. For the first step, a given velocity u is decomposed into a sum of series Intrinsic Mode Functions (IMF) $u(t) = \sum_{i=1}^N C_i(t) + r_N(t)$, in which $C_i(t)$ is IMF mode, and $r_N(t)$ is residual. In the second step, the classical Hilbert transform is applied to each mode to extract the instantaneous frequency ω. The original velocity is thus rewritten as $u(t) = \sum_{i=1}^N C_i(\omega, t)$, in which the residual is ignored. Therefore, the energy dissipation rate is rewritten as (the constant before the equation is left out)

$$\langle \left(\frac{\partial u}{\partial t} \right)^2 \rangle_t \equiv \underbrace{\int_0^{+\infty} \mathcal{A}(\omega) d\omega}_{\overline{\mathcal{E}}} = \sum_{i=1}^N \int_0^{+\infty} \underbrace{\langle \frac{\partial C_i(\omega,t)}{\partial t}^2 \rangle_t}_{\mathcal{B}(\omega)} d\omega + \sum_{i=1}^N \sum_{j=1, i\neq j}^N \int_0^{+\infty}\int_0^{+\infty} \underbrace{\langle \frac{\partial C_i(\omega_1,t)}{\partial t} \frac{\partial C_j(\omega_2,t)}{\partial t} \rangle_t}_{\mathcal{C}(\omega_1,\omega_2)} d\omega_1 d\omega_2 \tag{3}$$

in which $\mathcal{B}(\omega)$ is the dissipation density due to the movement of the eddy itself, $\mathcal{C}(\omega_1, \omega_2)$ is the redistribution of energy density between two different size of eddies.

EXPERIMENTAL RESULTS AND DISCUSSION

We consider here a velocity data base obtained from homogeneous and nearly isotropic turbulent flow with Reynolds number $Re_\lambda \simeq 720$ [4]. The measurement has been implemented at four locations $x/M = 20, 30, 40$ and 48, in which M is mesh size. Due to the absence of the spatial resolution of the Kolmogorov scale r_η, the energy dissipation rate $\overline{\epsilon}$ and Kolmogorov frequency f_η is estimated by an indirect way [4]. More detail about the experiment can be found in Ref. [4]. Figure 1 a) shows the measured $\mathcal{B}(\omega)$ at four measurement locations, in which the scale ω is normalized by the corresponding Kolmogorov frequency f_η and $\mathcal{B}(\omega)$ has been normalized by its first peak around $\omega/f_\eta \simeq 2$. Graphically, power law behavior is observed on the range $0.05 < \omega/f_\eta < 1$ with a scaling exponent $\gamma \simeq 0.37$ for all measurement locations. Note that a strong peak is observed around $\omega/f_\eta \simeq 30$, which is an effect of measurement noise. The total

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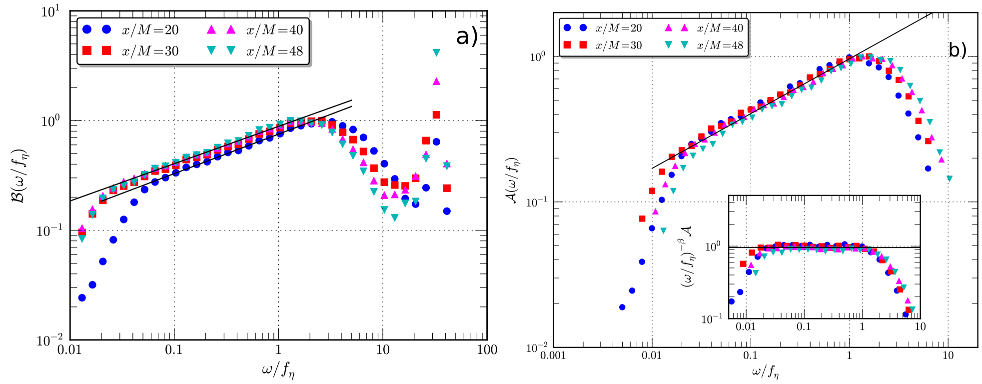

Figure 1. a) measured self energy dissipation density $\mathcal{B}(\omega)$, and b) the total energy dissipation density $\mathcal{A}(\omega)$. Both of $\mathcal{A}(\omega)$ and $\mathcal{B}(\omega)$ are normalized by the first peak and the scale ω is normalized by the Kolmogrov dissipation scale f_η. Power law behavior is observed. For the energy dissipation density the corresponding scaling exponent is 0.37.

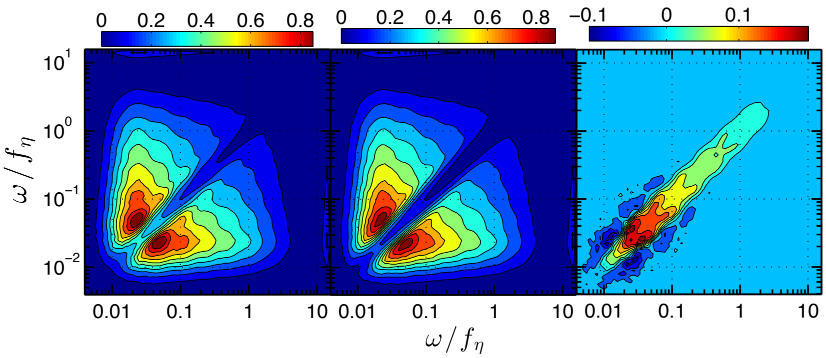

Figure 2. Measured energy redistribution density term $\mathcal{C}(\omega_1, \omega_2)$ at location $x/M = 20$ left, the positive part $\mathcal{C} > 0$, middle, negative part $\mathcal{C} < 0$ and right, the total $\mathcal{C}$. Note that the energy redistribution density is symmetric by the definition. The interaction is more strong in the low frequency part than high frequency part, indicating a nonlocal energy cascade.

energy dissipation rate $\overline{\epsilon}$ will be overestimated more than10 times if the measurement noise is included. To suppress the measurement noise, we remove the first IMF mode since it contains most of the measurement noise [3]. Figure 1 b) shows the total energy dissipation density function $\mathcal{A}(\omega)$ with denoising, in which the inset shows the compensated spectrum with a scaling exponent $\alpha = 0.37$, which is larger than the nonintermittent Kolmogorov value $1/3$. Note that a clear plateau is observed on the range $0.02 < \omega/f_\eta < 1$. This confirms the existence of the 0.37 power law, a similarity for the scale-based energy dissipation rate. This has been confirmed by other data base with $Re_\lambda \simeq 250$ and $Re_\lambda \simeq 3000$ (not shown here). Note that Eq. 1 indicates a value $\beta < 1/3$ if one considers the intermittent correlation. We argue here that the Eq. 1 is based on a linear decomposition, which is obviously not satisfied by the turbulent velocity, a typical nonlinear and nonstationary process [1, 3]. The Hilbert-based method we used here is suitable for nonlinear and nonstationary process by the meaning of Intrawave-Frequency-Modulation [2]. The measured $\mathcal{A}(\omega)$ reaches its maximum value around $\omega/f_\eta \simeq 2$. Furthermore, the dissipation below $\omega/f_\eta < 0.001$ and thus can be neglected. The dissipation from the inertial range is above 10% and must be taken into account when a turbulent model is proposed. It suggests that a 'real' direct-numerical-simulation should resolve the very fine scale as small as $r_\eta/10$.

To understand the energy redistribution between two different sale, we term the energy redistribution part $\mathcal{C}$ into two different part, positive one with $\mathcal{C}^+ > 0$ and negative one with $\mathcal{C}^- < 0$. The positive part $\mathcal{C}^+$ means the energy transfered from a given scale to other scales, and the negative one $\mathcal{C}^-$ means the energy gained from other scales. It is naturally to interpret the positive $\mathcal{C}$ as the energy dissipation during the energy redistribution process since the viscosity presents itself on all scales. Figure 2 shows the measured $\mathcal{C}^+$, $\mathcal{C}^-$ and $\mathcal{C}$. Graphically, the interaction does exist on a large range of scales, especially for the low frequency part (resp. large scale structures), indicating a nonlocal energy cascade. For a given scale, the interaction strength between nearby scales is stronger than others.

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