A MULTI-LAYER MODEL OF ROUGHNESS EFFECT IN TURBULENT PIPES

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Summary Turbulent rough pipe flow has been a theoretical challenge for nearly eighty years [1]. Notable recent findings include several ways of obtaining a single collapsed curve, called scaling function, for a series of roughness heights and Reynolds numbers (*Re*) [2]. Here, we extend a recently proposed multi-layer model [6,7,8] to characterize the roughness effects in terms of varying interaction between roughness elements and near-wall buffer layer structures in pipe flows. A multi-layer model is proposed for quantitative description of datasets of Nikuradze(1933) and Princeton rough pipe (2005) with good agreement.

INTRODUCTION

All practical pipes are rough, requiring understanding and quantification of roughness effects. The experiments carried out by Nikuradze (1933) [3] remain the classic benchmark - systematic measurement over a wide range of *Re*, with well-defined sand grains. Colebrook (1937) collected data for several industrial pipes [4], resulting in the construction of the widely-used Moody chart (1944). Most recently, the Princeton team (2006) [5] measured the honed pipe of relatively small roughness height at high *Re*. The most important effect of roughness is the change of the average velocity as a function of roughness element size, which then yields a variation of the friction factor. A quantitative description of this roughness dependent average velocity is still missing.

MULTI-LAYER INTERPRETATION

We have recently developed a quantitative multi-layer model for a smooth pipe, which yields an accurate prediction of the mean-velocity profile (MVP) [7] and mean kinetic-energy profile (MKP) [8]. This model gives a quantitative description of the so-called viscous sublayer and buffer layer, and also a reliable reference for smooth pipe flow at high *Re* for quantifying roughness effects. Fig.1 plots the difference in the section-averaged velocity between (measured) rough and (theoretical) smooth pipe, as a function of the roughness size normalized by the thickness of the (theoretical) viscous sublayer, for three sets of data mentioned above. All Nikuradze data and Princeton rough pipe data collapse to a single curve.

Next, the quantitative description of roughness effects by the multi-layer model divides into two stages:

First, denote k_e^+ as the roughness height (in wall units). Then, three regimes can be identified. 1) Hydraulically smooth regime when $k_e^+ < 1/\alpha$: The roughness effect is negligible, and the mean flow is hydraulically smooth; 2) Transitionally rough when $k_e^+ \geq 1/\alpha$: The roughness encroaches the buffer layer, and roughness-induced wakes modify near-wall vortices and streaks, and a new scaling arises; and 3) Fully rough when $k_e^+ >> 1/\beta$: The roughness induces its own near-wall motion, and so connect to the outer flow. While the first two regimes are sensitive to roughness size distribution, we expect that the third regime to be universal. Two characteristic heights, α and β, are introduced above, which must be determined from examination of the physics of the flow. We have the following consideration: 1) $\alpha \approx 5.5$ is a thickness determined by the near-wall structures such as streaks, beyond which significant roughness effect is felt. 2) $\beta \approx 150$ is the boundary of the bulk flow of developed three-dimensional structure, recently determined by She *et al.* [6].

Secondly, our smooth-pipe multi-layer theory predicts $\overline{U^+}^{Smooth} \approx (1/\kappa)\ln\left(\mathrm{Re}_\tau / y_*^+\right) + B(y_*^+)$ where $1/\alpha < y_*^+ < 1/\beta$ is a chosen near-wall characteristic distance. Accordingly, for transitionally rough case, we introduce a roughness-induced effective height $y_*^{+(rough)}$ such that $y_*^{+(rough)} / (\alpha k_e^+)^n = y_*^+$ defines an equivalent state in a smooth pipe: $\overline{U^+}^{Rough} \approx (1/\kappa)\ln\left(\mathrm{Re}_\tau / y_*^{+(rough)}\right) + B(y_*^+)$. With an exponent $n > 1$, we derive that $\overline{U^+}^{Smooth} - \overline{U^+}^{Rough} = (1/\kappa)\ln\left(\alpha k_e^+\right)^n$. For connecting with the hydraulically smooth state, we simply introduce $y_*^{+(rough)} / \left(1 + (\alpha k_e^+)^p\right)^{n/p} = y_*^+$ where p is a positive integer specifying the profile transition sharpness. This yields an analytic expression describing the first two

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roughness states: $\overline{U^+}^{Smooth} - \overline{U^+}^{Rough} = \frac{1}{\kappa}\ln\left(\left(1+(\alpha k_e^+)^p\right)^{n/p}\right)$. For fully rough regime, $k_e^+ >> 1/\beta$, $y_*^{+(rough)} \approx k_e^+$, independent of *Re*. The following model gives a complete description::

$$\overline{U^+}^{Smooth} - \overline{U^+}^{Rough} = \frac{1}{\kappa}\ln\left(\left(1+(\alpha k_e^+)^p\right)^{n/p}\left(1+(\beta k_e^+)^p\right)^{(1-n)/p}\right).$$

For large k_e^+, the r.h.s. tends to $(1/\kappa)\left(\ln k_e^+ + \ln\left(\alpha^n\beta^{1-n}\right)\right)$. Asymptotic universal state of roughness implies that . $\alpha^n\beta^{1-n} \approx \Gamma$, or $n \approx \ln(\Gamma/\beta)/\ln(\alpha/\beta)$. With $\alpha \approx 5.5$, $\beta \approx 150$, and $\Gamma \approx 2$, empirical data suggest that $n \approx 1.3$. Then, the scaling function for Nikuradze and Princeton rough pipe data is

$$f = \frac{1}{\kappa}\ln\left(\left(1+(\frac{k_e^+}{5.5})^4\right)^{1.3/4}\left(1+(\frac{k_e^+}{150})^4\right)^{-0.3/4}\right). \qquad (1)$$

The resulting friction coefficient formula for rough wall is

$$C_f^{Rough} = 8/\left(\frac{1}{0.45}\ln \mathrm{Re}_\tau + 4.1 - \frac{1}{0.45}\ln\left(\left(1+(\frac{k_e^+}{5.5})^4\right)^{1.3/4}\left(1+(\frac{k_e^+}{150})^4\right)^{-0.3/4}\right)\right)^2 \qquad (2)$$

Comparison with experimental data is shown in Fig.1; the agreement is excellent.

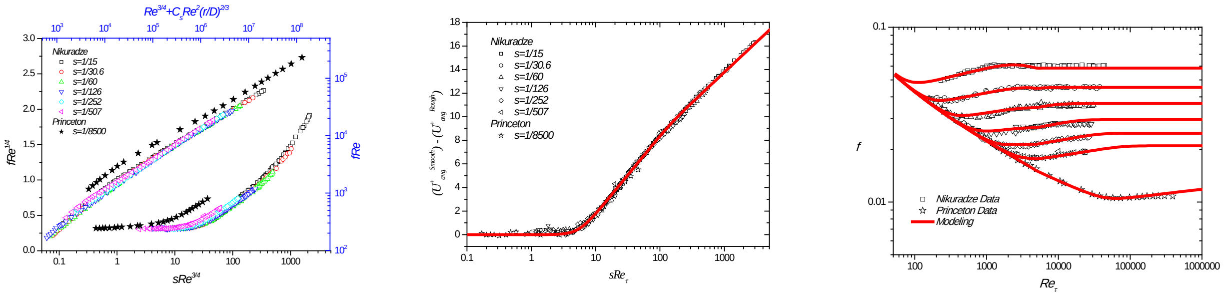

Figure 1. Nikuradze's sand-roughened pipes and Princeton honed pipe data collapsing according to Goldenfeld and Tao's schema (a), and the present schema (b). The line is the theoretical prediction of the multi-layer model for the scaling function (Eq.1) (b) and for the friction coefficient (Eq.2) (c).

DISCUSSION

For the first time, a complete collapse of the Nikuradze and Princeton rough pipe data is accomplished, thanks to a valid description of the smooth pipe at finite (moderate) *Re*. Then, a physical consideration of characteristic thicknesses of the multi-layer structure interacting with the roughness effect yields a quantitative description of the universal scaling function.

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