

REYNOLDS NUMBER EFFECTS ON THE SPECTRA FROM A TURBULENT FREE JET OF INITIAL SQUARE CROSS-SECTION AND THE KOLMOGOROV ISOTROPY "LAW"

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Summary The present study considers the influence of Reynolds number on a turbulent free jet issuing from a square duct. Detailed velocity measurements obtained using a single hot wire were performed at a jet exit Reynolds number spanning over the range of $8 \times 10^3 \leq Re_d \leq 5 \times 10^4$, where $Re_d \equiv U_j D_e / \nu$, where U_j is the centreline mean streamwise velocity at the exit plane, D_e is the equivalent diameter. Data were taken over axial distance of $0 < x/D_e < 40$. Compensated axial velocity spectra reveal a significant Re_d dependence on the width of the inertial sub-range. Plots of $f^m \Phi_U$ vs. $f Y_{1/2} / U_C$ where $Y_{1/2}$ and U_C are the local half-width and mean axial velocity indicate a preference for $1.40 < m < 1.56$ rather than $5/3$, which confirms earlier results from Antonia's group for round jets. The paper focuses on the effects of Reynolds number on the evolution of the mean and fluctuating velocities, the one-dimensional energy spectra and a variety of length scales associated with mixing transition.

INTRODUCTION

The study of turbulent jets remains a common topic in turbulence research. There are a number of studies that have explored the effects of Reynolds number on the spatial evolution of the mean and higher order statistics of jets that issue from round and non-round nozzles or orifices. Recently, Fellouah *et al.* [1] and Fellouah and Pollard [2] investigated the effect of Reynolds number on the near- and intermediate- fields region of a round nozzle jet using flying and stationary hot wire measurements. Their studies considered the velocity spectra and turbulence length scale distributions in the over the range of $6,000 \leq Re_d \leq 100,000$ to explore the idea of mixing transition as proposed by Dimotakis[3]. They found that the mixing transition is highly spatial and Re dependent. They also identified mixing transition through the emergence of an inertial subrange based upon compensated axial velocity spectra rather than the appearance of the transition from the inertial to dissipation ranges, as suggested by Dimotakis [3].

It is well known that inlet conditions affect higher order statistics in a round jet, see for example Hussain [4], George [5], Xu and Antonia[6], Uddin and Pollard[7]. A recent review article may also be consulted, Ball *et al.* [8]. From another perspective, the exit geometry of the jet flow has also been found to have an important influence on the behavior of jet flow. Noncircular jets have been found more effective in mixing with ambient fluid than comparable circular jets, and thus they have been investigated extensively in the literature. In the case of square jets in particular, it is known from large eddy simulation at laboratory scale Re numbers, that the vortex structure plays a significant role in the enhanced mixing associated with these jets relative to unforced round jets, Grinstein and DeVore [9] and Grinstein *et al.* [10]. The reason(s) for the enhanced mixing include the initial vortex structure at the exit of the square jet, which through simple Biot-Savart arguments give rise to collapse of the vortex in the corner regions and eventual axis switching; there are other spatial events that occur that present longitudinally oriented vortex structures, see Gutmark and Grinstein[11], which in the case of round jets (called braids) have been shown to increase entrainment and by inference the mixing, see Liepmann and Gharib [12], Citriniti and George [13] and McIlwain and Pollard [14].

In the case of square jets, there appear to be no data obtained for flow emanating from a long square tube, which gives rise to weak cross stream secondary flows induced by anisotropy of the Reynolds stresses. It is hypothesized here that these should alter the evolution of the jet, as they would instill an additional instability mechanism, and which is known to be Reynolds number dependent, see [15].

In this paper, data are presented that in the first instance characterizes the mean and Reynolds stress distribution in the near to intermediate field of a square jet that emanates from a long square duct with sharp corners over $8,000 < Re_d < 50,000$. These data are further processed to highlight changes in the compensated axial velocity spectra to show that the inertial subrange calculated according to isotropic, homogeneous turbulence emerges at a $Re=20,000$; however if scaling is altered from a $-5/3$ to $1.40 < m < 1.56$ exponent, as suggested by Mi and Antonia [16], the "inertial" range emerges earlier. The implications of this on the mixing transition are discussed.

BRIEF DESCRIPTION OF EXPERIMENTAL CONDITIONS

Velocity distributions of the jet flows issuing from a square duct were measured using single hot-wire anemometer. The duct that was 2.5 cm square and approximately 2m long. The nominal opening area $A=6.25\text{cm}^2$ and the equivalent diameter $D_e [\equiv 2(A\pi^{-1})^{1/2}]$ is $\sim 2.8\text{cm}$. The mean streamwise velocity U_j at the centre of the square exit plane was varied

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over the range $4.2 \leq U_j \leq 26.4$ m/s, corresponding to a Reynolds number $Re_d = 8 \times 10^3$ to 5×10^4 , where $Re_d \equiv U_j D_e / \nu$, with U_j the mean axial velocity at the jet exit. Measurements were performed along the centreline, $x/D_e \leq 40$. Velocity signals were low pass filtered with cut-off frequency $f_c = 20$ kHz. The effect of high frequency noise was removed using the method of Mi et al. [17].

RESULTS

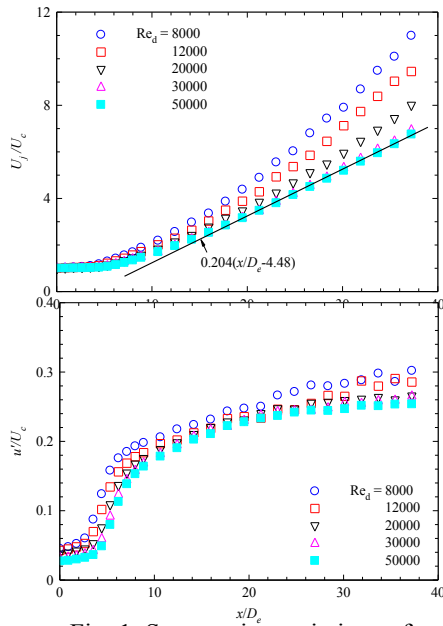


Fig. 1. Streamwise variations of (a) U_j/U_c ; (b) u'/U_c for $Re_d = 8000 \sim 50000$.

Fig. 1 shows the streamwise variations of inverse centerline mean velocity U_c normalized by the exit velocity U_j , i.e., U_j/U_c , half-widths $R_{1/2}/D_e$, the turbulence intensity u'/U_c for $Re_d = 8000 \sim 50000$. Fig. 1(a) demonstrates that the Re_d dependence of U_c is significant for $Re_d < 3 \times 10^4$ but weakens with increasing Re_d . Fig. 1(b) shows that u'/U_c grows rapidly in the initial region, which is caused by high local shear in the streamwise mean velocity. As x is increased further, u'/U_c increases gradually to an asymptotic value (K_u) in the far field. However, the magnitudes of u'/U_c are dependent on Re_d . Based on the averaged

value of u'/U_c over the region $30 \leq x/D_e \leq 40$, the far-field K_u is seen to first decrease with Re_d and then asymptote to $K_u \approx 0.25$ at $Re_d \geq 30000$.

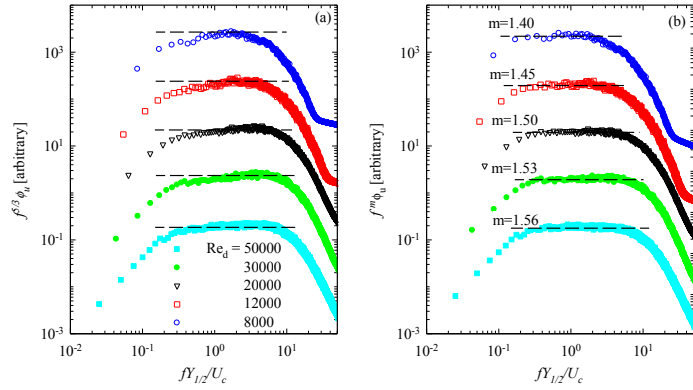


Fig. 2. Compensated velocity spectra $f^m \Phi_u$ of the centerline u measured at $x/D_e = 30$ for (a) $m=5/3$, and (b) $1.40 < m < 1.56$.

The compensated spectra $f^m \Phi_u$ are shown in Fig. 2. This figure indicates that all spectra exhibit a power-law region, i.e., $\Phi_u \propto f^{-m}$, over a certain range of f ; clearly, as expected, the range span widens as Re_d increases. However, the power-law exponent (m) is not Kolmogorov's '5/3', see figure 2(a), derived by assuming local isotropy, instead $1.40 < m < 1.56$ as displayed in figure 2(b). While the variation in m is perhaps expected as a function of Re , how these values are influenced by the intermittency, see [16] remains to be determined. This feature will be more fully addressed in the presentation.

CONCLUSIONS

The present study performed hot-wire measurements at $Re_d = 8 \times 10^3 \sim 5 \times 10^4$ to examine the Re_d dependence of the downstream development of the turbulent jet from a square duct or pipe. Within this Re_d range, significant dependences are found in the mean velocity decay rate (and half-width, not shown), the evolution of turbulent intensity as well as the scaling parameter for the compensated spectra.

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