# TURBULENT MODELING OF WALL-BOUNDED FLOW WITH MEMORY EFFECTS[A)]

Xiang Qiu[*b)], G. Mompean[**], F. G. Schmitt[***] & R.L. Thompson[****]
[*] *School of Science, Shanghai Institute of Technology, Shanghai 200235, China*
[**] *USTL, LML, F-59655 Villeneuve d'Ascq, France; CNRS, UMR 8107, F-59655 Villeneuve d'Ascq, France;*
[***] *USTL, LOG, F-62930 Wimereux, France; CNRS, UMR 8187, F-62930 Wimereux, France;*
[****] *UFF, LMTA , Niterói RJ24210-240, Brazil*

*Summary* Memory effects have been recognized to be very important in turbulent shear flows. And turbulent flows of Newtonian fluids have already been compared with non-Newtonian laminar flows. In this paper the analogy between these classes of flows is explored, and a new approach to derive a turbulent model concerning memory effects based on a nonlinear constitutive equation is shown. In order to reach this aim, opened DNS databases of turbulent channel flows are used and analyzed in the light of the classical parameters of non-Newtonian constitutive equations. The Reynolds stress tensor is expressed in terms of a set of basis tensors based on a projection of a nonlinear framework. The coefficients of the model are given as functions of the intensity of the mean strain tensor. The apparent turbulent viscosity, the first and second normal stress difference, are presented in function of the shear rate. These material functions, exhibiting a shear-thinning behavior, are fitted with the power law (the Carreau-type) model. The range of the Reynolds number investigated was $180 \leq Re_\tau \leq 2000$. One of the advantages of the new algebraic nonlinear power law constitutive equation derived in the paper is that its dependence is only on the mean velocity gradient and can be integrated up to the wall.

## INTRODUCTION

Turbulence exist widely in nature and engineering, it is a classical complicated nonlinear problem. Owing to the long decay process of turbulent large scale structures, turbulent quantities could not be presented only by local quantities, and the history process must be considered in turbulent modeling, so to develop new turbulent model interrelated to memory effects are significant to enrich turbulent theory and exploit engineering application.

The analogy between the turbulent Reynolds stress tensor for Newtonian fluids and the constitutive laws for viscoelastic flows is explored in this paper. Rivlin (1957) was probably the first one to qualitatively investigate the relation between the laminar flows of non-Newtonian fluids and the turbulent flows of Newtonian fluids. Later, such similarities were explored in several other papers Townsend (1966); Crow (1968); Groisman & Steinberg (2000).

The purpose of this work is to propose an approach, alternative to the traditional $k-\varepsilon$ one, to capture the coefficients of the non-linear model expressed by a three-tensor basis that generalizes the Boussinesq hypothesis. Based on the analogy with non-Newtonian fluids, these coefficients are related to the viscosity, first and second normal stress coefficients, which are usual material functions obtained from shear flows of viscoelastic fluids.

## NONLINEAR CONSTITUTIVE EQUATIONS AND VISCOMETRIC FUNCTIONS

Let us denote $\mathbf{R}_{ij} = \frac{2}{3}\rho\delta_{ij}k - \rho\overline{u_i u_j}$ as the anisotropic traceless stress tensor. $k$ is the turbulent kinetic energy ($k = \frac{1}{2}\overline{u_i u_i}$) and $\delta_{ij}$ the Kronecker delta function.In the 2-D framework, the anisotropic tensor **R** can be written as follows(Bird 1987),

$$\frac{\mathbf{R}}{\rho} = 2\nu_T\mathbf{S} - \beta(\mathbf{SW} - \mathbf{WS}) - \gamma(\mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I}) \tag{1}$$

where the coefficients $\nu_T$, $\beta$ and $\gamma$ are given as functions of the basic invariants of the flow (Schmitt 2007),

$$\nu_T = \frac{\{\mathbf{RS}\}}{\{\mathbf{S}^2\}} = \frac{-\overline{uv}}{a}, \beta = \frac{\{\mathbf{RSW}\}}{\{\mathbf{S}^2\}\{\mathbf{W}^2\}} = \frac{\overline{uu} - \overline{vv}}{a^2}, \gamma = \frac{-6\{\mathbf{RS}^2\}}{\{\mathbf{S}^2\}^2} = \frac{6}{a^2}(\frac{2}{3}K - \overline{ww}), \tag{2}$$

where $\{\}$ is the trace operator, $\overline{uv}, \overline{uu}, \overline{vv}$ and $\overline{ww}$ , the shear and the three normal Reynolds stress components.

In order to go further, we consider viscoelastic analogy, since viscoelastic flows possess normal stress differences and the viscosity is shear dependent. Here we introduce viscometric functions into turbulent modeling. The mechanical properties of the flow are fully determined when three functions are known, $\tau(a)$, $\Psi_1(a)$, $\Psi_2(a)$, shear, first and second normal stress difference, respectively. These functions are expressed against the strain rate variable $a$ as follows (Qiu et al 2011)

$$\nu_T(a) = \frac{-\overline{uv}}{a} = \frac{\tau(a)}{\rho a}, \beta(a) = \frac{\overline{uu} - \overline{vv}}{a^2} = \frac{\Psi_1(a)}{\rho}, \gamma(a) = \frac{6}{a^2}(\frac{2}{3}K - \overline{ww}) = \frac{2}{\rho}[\Psi_1(a) - 2\Psi_2(a)]. \tag{3}$$

For an unknown viscometric flow, these functions are experimentally estimated and some general properties of the flow are inferred. For Newtonian flows $\tau(a)/a$ is constant, so if this ratio depends on $a$, the flow possesses non-Newtonian (e.g. shear thinning) characteristics. To explore the analogy, we assume here that the total stress of a turbulent shear flow corresponds to a viscometric flow, and see what can be said about the different viscometric functions.

[a)] Project supported by the NSFC (No.11102114,11172179 ).
[b)] Corresponding author. E-mail: emqiux@gmail.com.

Using the expression of total stress $\mathbf{R}_{total} = \mathbf{R}_v + \mathbf{R}$, then the quadratic constitutive equation for the total stress can be written in terms of the viscometric functions as

$$\frac{\mathbf{R}_{total}}{\rho} = 2(\nu + \nu_T)\mathbf{S} - \frac{\Psi_1(a)}{\rho}(\mathbf{SW} - \mathbf{WS}) - \frac{2}{\rho}[\Psi_1(a) - 2\Psi_2(a)](\mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I}), \quad (4)$$

## DATABASES AND TURBULENT MODELING

We consider here several DNS databases of shear flows, characterized by a range of Reynolds numbers $Re_\tau$ from 180 to 2000. These databases correspond to plane channel flow and Poiseuille flow, see details from Qiu et al 2011.
In order to use the DNS data base to derive the material functions, the following non-dimensionalisation is introduced :

$$y^+ = \frac{y}{y_0}, \quad U^+ = \frac{U}{u_\tau}, \quad \tau^+ = \frac{\tau}{\rho u_\tau^2}, \quad a^+ = \frac{dU^+}{d(y/\delta)} = a\frac{\delta}{u_\tau} \quad (5)$$

where $u_\tau = \sqrt{\tau_w/\rho}$ is the characteristic velocity, $y_0 = \nu/u_\tau$ the characteristic length, and $\delta$ is half width of the channel.

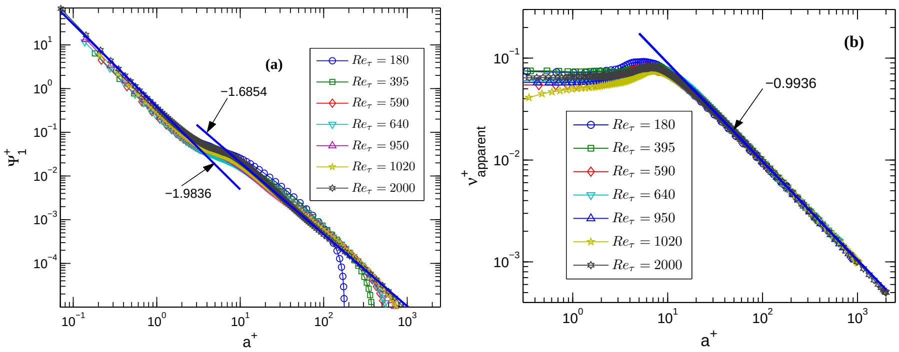


**Figure 1.** Profiles of (a) viscometric function $\Psi_1$ and (b) apparent viscosity $\nu_{app}$ vs. shear rate $a^+$.

Using the DNS data presented in Fig.1(b) to obtain $\nu_{app}$ , we can write the model as follows

$$\nu^+_{app}(a^+) = \nu^0_{app}[1 + (\lambda^+_{c0}\ a^+)^2]^{\frac{n_0-1}{2}}, where\ \ \nu^0_{app} = 0.0655, \quad n_0 = 0.0064, \quad \lambda^+_{c0} = 0.0678. \quad (6)$$

For the first viscometric function $\Psi_1^+$, since there are two slopes (see Fig.1(a)), one for the small $a^+$ region and and one for the large $a^+$ region, the following expression to fit $\Psi_1^+$ is used:

$$\Psi_1^+(a^+) = \psi_1 \mathrm{exp}(-\lambda_{c1}\ a^+)(a^+)^{(m_1-1)} + \psi_2[1 - \mathrm{exp}(-\lambda_{c2}\ a^+)](a^+)^{(n_1-1)} \quad (7)$$

For small $a^+$ ($a^+ < 4$) $\Psi_1^+$ tends to be $\psi_1(a^+)^{(m-1)}$ , and for larger $a^+$ , $\Psi_1^+$ tends to be $\psi_2(a^+)^{(n-1)}$. The coefficients of the model can be obtained as below:

$$\psi_1 = 0.3501, \quad \psi_2 = 1.0957, \quad \lambda_{c1} = 0.2, \quad \lambda_{c2} = 0.11, m_1 - 1 = -1.9836, \quad n_1 - 1 = -1.6854. \quad (8)$$

For the viscometric function $\Psi_2^+$ (figure not shown here), also using the same previous approach for $\Psi_1$, we get:

$$\Psi_2^+(a^+) = \phi_1 \mathrm{exp}(-\lambda_{d1}\ a^+)(a^+)^{(m_2-1)} + \phi_2[1 - \mathrm{exp}(-\lambda_{d2}\ a^+)](a^+)^{(n_2-1)} \quad (9)$$

For small $a^+$ ($a^+ < 4$), $\Psi_2^+$ tends to be $\phi_1(a^+)^{(m-1)}$, and for larger $a^+$, $\Psi_2^+$ tends to be $\phi_2(a^+)^{(n-1)}$. Based on the two slopes, we have the following coefficients:

$$\phi_1 = 0.0117, \quad \phi_2 = 0.2605, \quad \lambda_{d1} = 1, \quad \lambda_{d2} = 0.05, m_2 - 1 = -1.9434, \quad n_2 - 1 = -1.6760. \quad (10)$$

For this model,non-Newtonian material functions were introduced to derive the parameters $\nu_{apparent}$, $\beta$ and $\gamma$, instead of using the turbulent kinetic energy and dissipation rate scales in classical turbulent models, as $k - \epsilon$ model.

## References


[1] Rivlin R.S.: The relation between the flow of non-Newtonian fluids and turbulent Newtonian fluids. *Quart Appl. Math.*, **15**: 212–214, 1957.
[2] Townsend A.A.: The mechanism of entrainment in free turbulent flows. *J. Fluid Mech.*, **26**: 689–715, 1966.
[3] Crow S.C.: Viscoelastic properties of fine-grained incompressible turbulence. *J. Fluid Mech*, **33**: 1–20, 1968.
[4] Groisman A., Steinberg V.: Elastic turbulence in a polymer solution flow. *Nature*, **405**: 53–55, 2000.
[5] R.B. Bird, R.C. Armstrong, O. Hassager.: Dynamics of Polymeric Liquids. *Wiley, New York, In: (2nd ed.)*, **I**, 1987.
[6] Schmitt F. G.: Direct test of a nonlinear constitutive equation for simple turbulent shear flows using DNS data. *Comm. Nonlin. Sc. Num. Sim.*, **12** :1251–1264,2007.
[7] Qiu X., Mompean G., Schmitt F.G., Thompson R.L.: Modeling turbulent bounded flow using non-newtonian viscometric functions. *Journal of Turbulence* **12(15)**: 1–18, 2011.