RECONNECTION OF VERY LONG VORTEX PAIRS WITH A TRANSITION TO TURBULENCE

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Summary It is found that by initialising with very long counter-rotating vortices, and a very localised perturbation, that a series of Navier-Stokes reconnection events develop and out of this turbulent statistics are generated. The mechanism of stretching and twisting is analogous to the Lundgren spiral. The importance of using such long vortices, compared to earlier work with similar initial conditions, is that the stretching that drives the later evolution does not come from the initial reconnection on the symmetry plane of the initial perturbation, but from the subsequent reconnection events. The dynamics quickly generate a -5/3 energy spectrum and normalised vortex stretching rates consistent with isotropic turbulence, even though the flow is far from isotropic. Analysis of higher-order structure functions and vorticity moments is ongoing and will be discussed.

Short paper. Vortex interactions and reconnection have often been proposed as mechanisms for generating the small scales for turbulence, generating singularities, and the origins of the turbulent energy cascade. However, up to now, numerical studies of reconnection between two vortex tubes have fallen short of these goals. Turbulence has been generated by numerical simulations, but only by using random initial conditions, long-time forcing, or collisions of many vortices reconnecting many times. And attempts to determine regularity properties of the inviscid Euler equations have proven ambigious and controversial. Based upon recent discussions [1], it has become clear that part of the problem has been the initial conditions used as the small-scale evolution has been difficult to reproduce using different numerical codes.

Since some of these issues appear to be unique to Euler flows, where by Kelvin's theorem the circulation must be rigorously conserved, perhaps more could be learned by using and improving upon these initial conditions for systems where the circulation is not rigorously conserved and where we either know, or believe, that the underlying equations are regular. The results of two such systems would be reported in this presentation.

The first set of results, reconnection in the Gross-Pitaevskii equations, used for superfluids and Bose-Einstein condensates, have been published [2] and suggest that the decay of *quantum turbulence*, a tangle of quantum vortex lines, is due to reconnection of the lines into vortex rings, which then propagate out of the system. As originally suggested by Feynmann (1956). These results are currently being extended in collaboration with the Maryland group to help us understand recent visualisations of superfluid vortex reconnection [3].

The primary focus of this presentation would be to extend this success in relating the quantum vortex reconnection to the formation of quantum turbulence to reconnection and turbulence development in the Navier-Stokes equations. While in many respects the initial condition used for the new calculations is similar to older work [4, 5], there are many improvements based upon new trajectories for the tubes [2] and a new profile based upon exact balanced solutions for pairs of quantum vortices [6]. This class of initial conditions is needed in the quantum case in order to get clean results that reproduce the experimental scaling laws for how vortices approach, reconnect, and separate. The reasons are similar to why a balanced, barotropic initial condition is needed for compressible Navier-Stokes vortices [7]. Details of the new initial condition will be presented at the meeting.

The simulation discussed here is a pseudo-spectral calculation of the incompressible Navier-Stokes equations. The symmetries of the initial vorticity configuration are used so that only 1/4 of the full domain needs to be simulated. The simulated domain is $L_x \times L_y \times L_z = 2\pi[2 \times 8 \times 1]$ and the highest Reynolds number (used here only for the spectra) uses $n_x \times n_y \times n_z = 512 \times 2048 \times 512$ mesh points and its initial Reynolds number is $\Gamma/\nu \approx 4000$, where Γ is the initial circulation in the $x - z$ symmetry plane. If the vorticity is $\boldsymbol{\zeta} = \nabla \times \boldsymbol{u}$, then the enstrophy is $Z = \overline{\boldsymbol{\zeta}^2}$ and the Lagrangian enstrophy density equation is $0.5D\zeta^2/Dt = \zeta_i S_{ij}\zeta_j + 0.5\nu\zeta_j\nabla^2\zeta_j$ where S_{ij} is the strain. The isotropic velocity derivative skewness $-S_u$ discussed below can be formed from $\overline{\zeta_i S_{ij}\zeta_j}$ normalised by $Z^{3/2}$.

With this new initial condition, two problems that have plagued many earlier Navier-Stokes reconnection calculations do not appear [1]. First, there is no unphysical negative vorticity on the symmetry plane. Second, the first reconnection event results in two compact orthogonal vortices, each with about half the original circulation [7], and does not leave behind addition braids and vortex sheets.

The orthogonal property is essential for what happens next. In a manner similar to recent observations of the helical vortex interactions [8], these vortices spiral around each other and quickly generate small scales, as predicted by the Lundgren spiral.

However, one reconnection is not enough. As in the quantum case [2], where very long vortices were needed for rings to form, the stretching that leads to the first symmetry plane reconnection is not enough to maintain strong vortex stretching. Another source of stretching is needed, and this is supplied by a set of secondary reconnections away from the symmetry plane. In the quantum case this leads to the formation of a pair of vortex rings, which then remove energy from the system by propagating away. In the Navier-Stokes case, dissipation from their internal spiral motion takes over before the rings can fully form.

Is this spiral motion enough to generate turbulent statistics? Not quite. A very long domain is then needed so that a chain reaction of reconnection events can form, and it is from this, if the Reynolds number is sufficiently high, that turbulent statistics, including a -5/3 spectrum, appear.

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What about higher-order statistics? Analysis is in progress on structure functions. The third-order structure function has a linear regime, but the magnitude is about half the isotropic prediction, perhaps because most of the dissipation is still bound in the original vortices. Further work on higher-order structure functions is needed to resolve this question. Even more promising is the behaviour of the higher-order vorticity moments. Analysis of the new numerical data supports new analytic work [9], which discusses the intermittency properties of the vorticity moments $\tilde{\Omega}_m(t) = (L^{-3}\int dV\omega^{2m})^{1/2m}/\varpi$ scaled by the characteristic frequency $\varpi = \nu L^{-2}$. The *roughness in time* of these quantities closely follows the temporal variations in $-S_u$ shown below.

Conclusions. The captions of the following figures summarise these results.

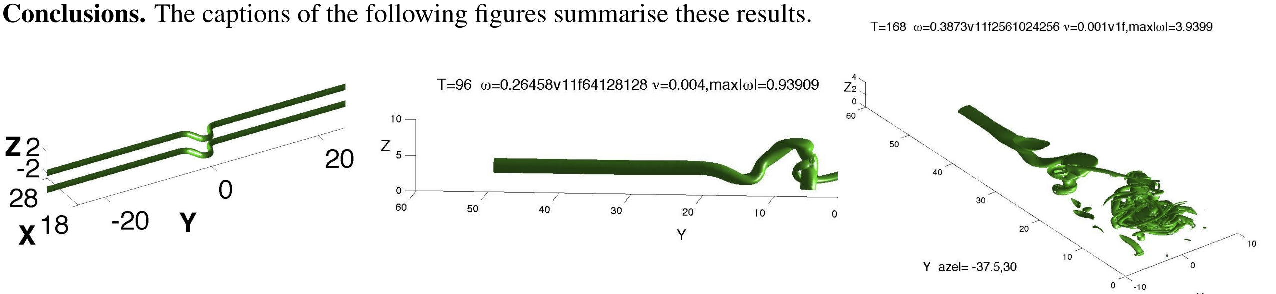

Figure 1: Evolution of isosurfaces of vorticity. Left: Original vortices in the full domain. Others: One half of one of the vortices. Middle: t=96: Note reconnected orthogonal vortices on right and large loop due to the second reconnection forming near y=12. Right: t=168: The ring has turned into the turbulence on the right, a second spiral is forming near y=40 and a third reconnection is forming near y=70. This is related to the spectra below.

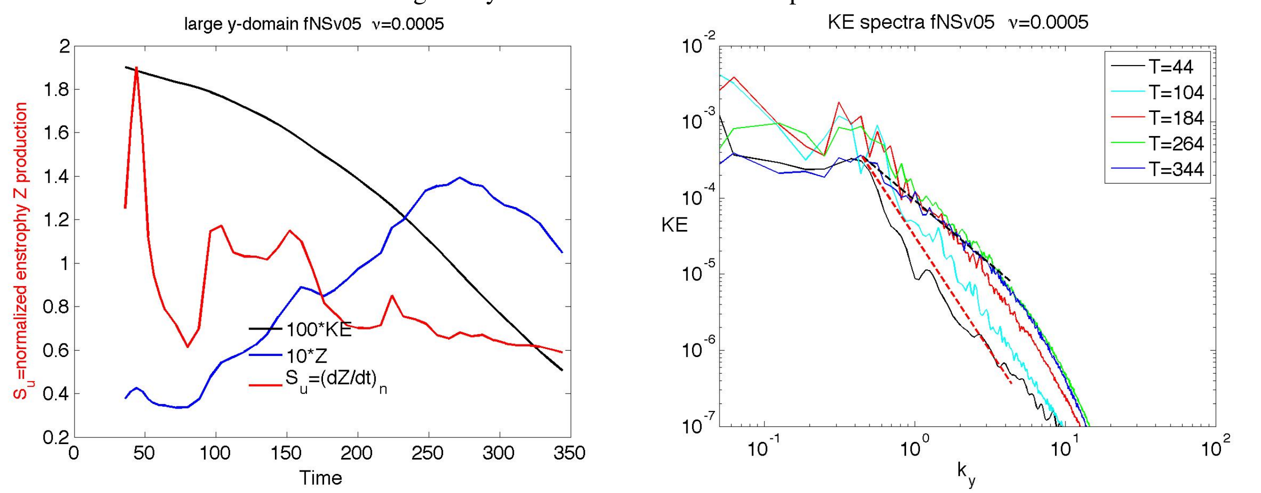

Figure 2: Left: Classical evolution properties: Kinetic energy decay, growth then decay of the enstrophy, and normalised enstrophy production, written as the velocity derivative skewness $-S_u$. $-S_u$ sets the timescales for the figures above. Its first peak near $t = 40$ is when the first reconnection has completed and the second peak near $t = 90$ is due to the stretching induced by the formation of the second reconnection shown above. Ongoing analysis is showing that the major temporal fluctuations in $-S_u$ are closely associated with fluctuations of higher-order moments of the enstrophy [9]. Right: In this crowded preliminary plot of spectra, one will see that a -5/3 spectrum first appears at $t = 184$, after the skewness has settled into $-S_u \sim 0.5 - 06$, the isotropic turbulence value. The last frame in Fig. 1 shows that at roughly this time, the first spiral from the first reconnection has become locally turbulent, even though the overall flow is still far from being isotropic and homogeneous. While a larger -5/3 span appears later, at $t = 344$, once turbulence has filled the entire domain and the peak in enstrophy has passed, the earlier time shows that -5/3 can also form from a twisted spiral as suggested by Lundgren.

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