Moving Particle Semi-implicit Method Based on Large Eddy Simulation

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Summary Turbulence model is necessary for MPS method because most free surface flows in engineering are turbulent flows. Taken idea from the sub-grid stress in the LES, a sub-particle stress based on the Smagorinsky model is developed for the MPS method. The first order derivative in the Smagorinsky model is discretized by the least square method to improve the accuracy and stability, and the sub-particle stress is solved in the explicit part of MPS. The results of the dam breaking problem simulated by the developed method are in good agreement with the experimental observations.

INTRODUCTION

The Moving Particle Semi-implicit method (MPS) was proposed by Koshizuka [1] to simulate the incompressible free surface flow. MPS is a fully Lagrange method which makes it easy to capture the free surface boundary and calculate the advective term. Most industrial free surface flows are turbulent, so it's of great significance to develop turbulence model for the MPS method. LES is chosen to be coupled with MPS since it is compromise between DNS and RANS.
The reference studies this topic is few because MPS is not very mature currently. In references [2-4], the first order derivative in Smagorinsky model is discretized by mimicking the gradient model, which is inaccurate near the free surface. In this paper, the Smagorinsky model is discretized by the Moving Least Square method (MLS) to improve the stability and accuracy of the coupled method. The dam breaking model is simulated to test the effect of the method.

THE METHOD

In this section, the MPS-LES method is described by two parts: the Smagorinsky model and the discretization.

1. Smagorinsky model

When the N-S equations are filtered in the physical space, a new Sub-Filter Stress term, which is addressed as Sub-Particle Stress (SPS) in particle method, will emerge in the momentum equation. SPS can be modeled by the widely used Smagorinsky model[5] as follows,

$$\bar{\tau}_{ij} = 2\nu_t \bar{S}_{ij} - \frac{2}{3}k\delta_{ij} = \nu_t\left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i}\right) - \frac{2}{3}k\delta_{ij} \quad (1)$$

where $\bar{S}_{ij}$ is the resolved strain-rate tensor, ν_t is the eddy viscosity and k is the turbulent kinetic energy. The model above is a theoretical one, which can be applied to both conventional grid method and particle method.

2. Discretization of SPS model

MPS is a fully Lagrange method, so the fluid is discretized as fluid particles, which are similar to the portion of fluid. The MPS method discretizes the derivative operators by the particle interaction models as follows,

$$\langle \nabla\varphi \rangle_i = \frac{d}{n_0}\sum_{j\neq i}\frac{(\varphi_j - \varphi_i)(\mathbf{r_j} - \mathbf{r_i})}{|\mathbf{r_j} - \mathbf{r_i}|^2} w(|\mathbf{r_j} - \mathbf{r_i}|) \quad (2)$$

$$\langle \nabla^2\varphi \rangle_i = \frac{2d}{n_0\lambda}\sum_{j\neq i}(\varphi_j - \varphi_i) w(|\mathbf{r_j} - \mathbf{r_i}|) \quad (3)$$

where φ is a general scalar variable, $w(r)$ is the weight function and n_0 is the initial constant particle number density. One shortcoming of the models above is that the model will be inaccurate when the reference particles are near the free surface. The turbulence effect mainly concentrates on the free surface, so MLS, which is more accurate, is adopted to discretize the Smagorinsky model to improve the accuracy of the turbulent model. The MLS model used here is based on the Multivariate Taylor series as follows,

$$\frac{\varphi_j - \varphi_i}{\rho} = \frac{\partial\varphi_i}{\partial x}\cos(\theta) + \frac{\partial\varphi_i}{\partial y}\sin(\theta) + \frac{\rho}{2}\left[\frac{\partial^2\varphi_i}{\partial x^2}\cos^2(\theta) + 2\frac{\partial^2\varphi_i}{\partial x\partial y}\cos(\theta)\sin(\theta) + \frac{\partial^2\varphi_i}{\partial y^2}\sin^2(\theta)\right] \quad (4)$$

where j is the label of neighbor particles of the reference particle i, $\rho = \sqrt{\Delta x^2 + \Delta y^2}$, $\cos(\theta) = \frac{\Delta x}{\rho}$, $\sin(\theta) = \frac{\Delta y}{\rho}$.

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DAM BREAKING SIMULATION

The dam breaking problem is simulated by present method. Three schemes are used to simulated dam breaking problem. Both the molecular viscosity term and eddy viscosity term are neglected in the "Inviscid" scheme. Only the turbulent eddy viscosity term is neglected in the "Viscous" scheme. All the terms in Eqs (1) are considered in the "LES" scheme. The simulated results of different schemes are shown in Fig. 1, which shows that "LES" scheme is in more satisfactory agreement with the experiment data. The turbulent kinetic energy can show the intensity of turbulent effect. The turbulence mainly concentrates on the front of water column as shown in Fig. 2.

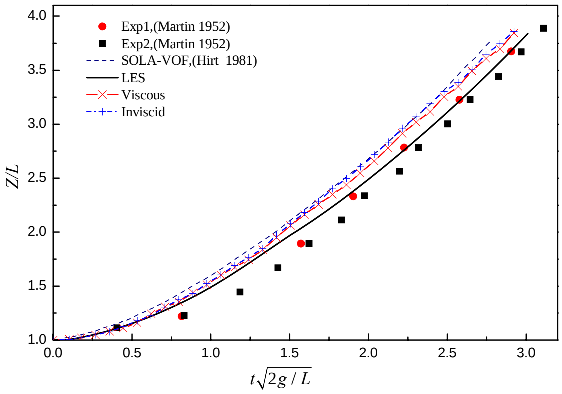

Fig. 1 The comparison between simulated and experimental results in the dam-break model

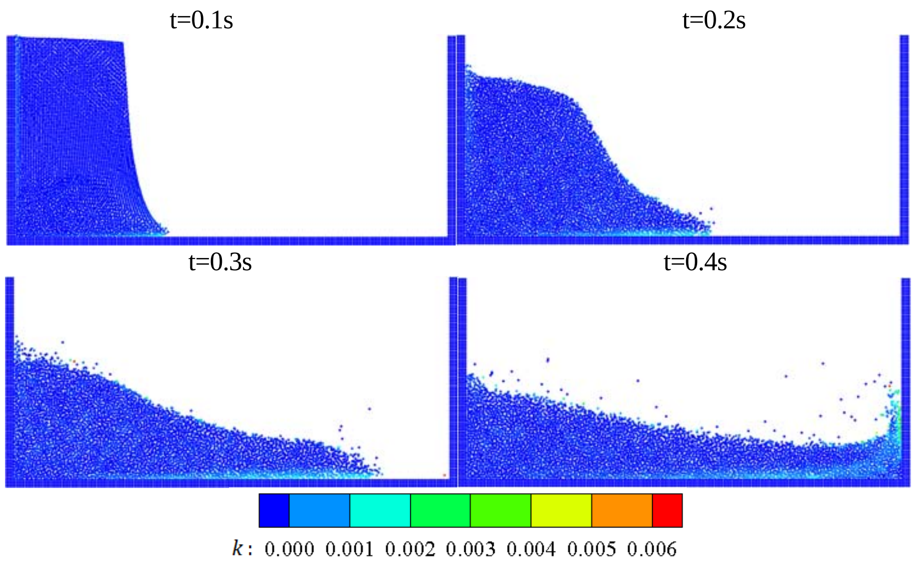

Fig. 2 The turbulent kinetic energy distribution at four different moments

CONCLUSIONS

The Smagorinsky model is coupled in the MPS method, so the turbulence effect is considered in simulating the free surface flow by MPS. The key issue in the coupling is how to discretize the Smagorinsky model. To avoid the accuracy degradation of the origin MPS model near the free surface, moving least square method is adopted to discretize the Smagorinsky model, which is calculated in an explicit way. The simulated result includes that the Smagorinsky model is in better agreement with experimental data than the results neglecting the turbulence effect. Furthermore, the effect of free surface and solid boundary on turbulence will also be considered.

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