NORMALIZATION OF THE MEAN RATE OF ENERGY DISSIPATION IN TURBULENCE

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Summary The mean rate of energy dissipation $\langle\varepsilon\rangle$ in turbulence is often written in the form of $\langle\varepsilon\rangle = C_u\langle u^2\rangle^{3/2}/L_u$, where $\langle u^2\rangle^{1/2}$ is the root-mean-square fluctuation of the longitudinal velocity u and L_u is its correlation length. However, the dimensionless coefficient C_u is known to depend on the large-scale configuration of the flow. We define the correlation length L_{u^2} of the local energy u^2 and find a possibility that $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ does not depend on the flow configuration.

INTRODUCTION

Since the kinetic energy of turbulence is transferred from large to small scales and is eventually dissipated into heat, the mean rate of the energy dissipation per unit mass $\langle\varepsilon\rangle = 15\nu\langle(\partial_x u)^2\rangle$ is independent of the value of the kinematic viscosity ν. This rate is instead determined by parameters of the large scales:

$$\langle\varepsilon\rangle = C\frac{\langle u^2\rangle^{3/2}}{L}. \tag{1}$$

Here C is a dimensionless coefficient, $\langle u^2\rangle^{1/2}$ is the root-mean-square fluctuation of the longitudinal velocity u, and the length L represents the large scales or equivalently the sizes of the energy-containing eddies. The energy of such eddies is of the order of $\langle u^2\rangle$. Their time scale is of the order of $L/\langle u^2\rangle^{1/2}$. As a result, $\langle u^2\rangle^{3/2}/L$ is of the order of the mean rate at which their energy is transferred to the smaller eddies. Following the pioneering works of Taylor [1] in 1935 and Batchelor [2] in 1953, Eq. (1) has been used in various studies of turbulence.

The length L is traditionally defined as the correlation length L_u of the longitudinal velocity u, which is based on the two-point correlation $\langle u(x+r)u(x)\rangle$:

$$L_u = \frac{\int_0^\infty \langle u(x+r)u(x)\rangle dr}{\langle u^2\rangle}. \tag{2}$$

However, by using experimental and numerical data from the literature, Sreenivasan [3, 4] found that $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ depends on the flow configuration. That is, C_u is determined by the initial condition, boundary condition, and external force that induce the turbulence. For the same flow configuration, C_u was found to be a constant of $O(1)$ if the Reynolds number is high enough to separate the energy-containing large scales from the energy-dissipating small scales. These findings have been confirmed experimentally [5] and numerically [6].

The dependence of C_u on the flow configuration implies that $\langle u^2\rangle^{3/2}/L_u$ is not proportional to the mean rate at which the energy is removed from the energy-containing eddies. It is hence concluded that L_u is not proportional to the typical size L of the energy-containing eddies, especially in terms of their time scale $L/\langle u^2\rangle^{1/2}$.

This conclusion is important because L_u has been used as a representative of the large scales, not only in the study of Eq. (1) but also in many other studies. It is desirable to explore a more suitable definition of L. Here, we define L as the correlation length L_{u^2} of the local energy u^2:

$$L_{u^2} = \frac{\int_0^\infty \langle[u^2(x+r)-\langle u^2\rangle][u^2(x)-\langle u^2\rangle]\rangle\, dr}{\langle(u^2-\langle u^2\rangle)^2\rangle}. \tag{3}$$

By using experimental data of several flows, $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ is studied over a range of the Reynolds number $Re_\lambda = \langle u^2\rangle^{1/2}\lambda/\nu$ for the Taylor microscale $\lambda = [\langle u^2\rangle/\langle(\partial_x u)^2\rangle]^{1/2}$.

EXPERIMENTAL DATA

The experimental data used here are those of the streamwise velocity u obtained in a wind tunnel. They are for grid turbulence G1–G5 (Re_λ = 153–436), boundary layer B1–B6 (Re_λ = 455–2097), and jet J1–J6 (Re_λ = 709–3315), among each of which the flow configuration was the same. While B1, B4, B6, J1, and J6 are used here for the first time, the others were used in our past works [7, 8]. These data have been obtained and processed in the same manner. They are also long enough so that the resulting statistics are reliable. Figure 1 compares $\langle u(x+r)u(x)\rangle$ with $\langle[u^2(x+r) - \langle u^2\rangle][u^2(x)-\langle u^2\rangle]\rangle$. Since the u^2 correlation decays faster than the u correlation, L_{u^2} is smaller than L_u.

RESULTS

Figure 2 shows $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ and $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ as a function of Re_λ. Since Re_λ is in a log scale, it is emphasized that C_u and C_{u^2} are not yet constant but decrease with an increase in Re_λ. The same tendency is seen in

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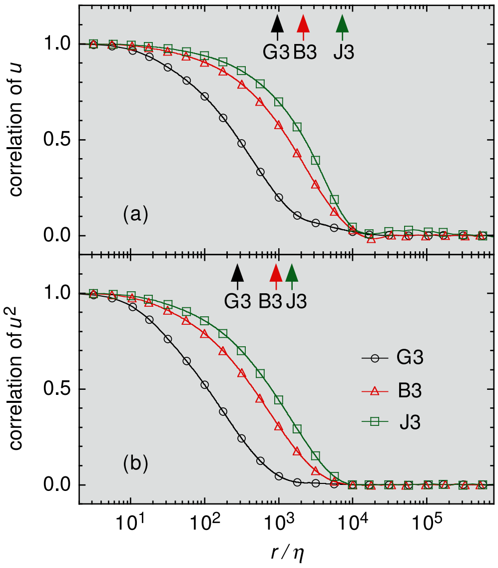

Figure 1. Two-point correlations of u and u^2 as a function of scale r normalized by the Kolmogorov length $\eta = (\nu^3/\langle\varepsilon\rangle)^{1/4}$ in grid turbulence G3 (◯), boundary layer B3 (△), and jet J3 (□). The correlations are normalized by their values at $r = 0$. The arrows indicate L_u and L_{u^2}.

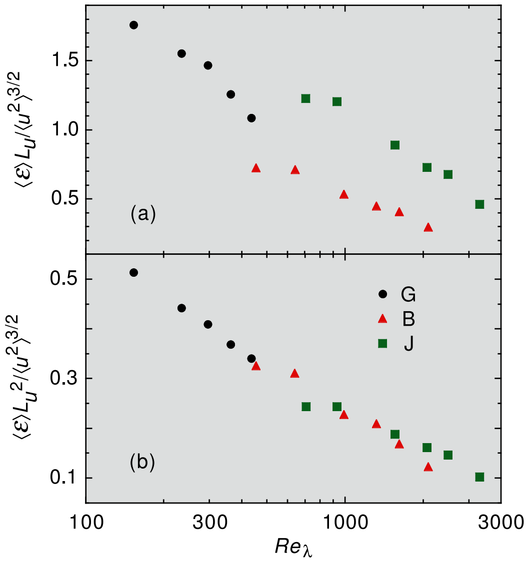

Figure 2. Normalized rates of energy dissipation $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ and $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ as a function of Re_λ in grid turbulence G1–G5 (●), boundary layer B1–B6 (▲), and jet J1–J6 (■).

results of the past works [5, 6]. These values of Re_λ are not high enough for complete separation of the large scales from the small scales [9]. More importantly, among the grid turbulence, boundary layer, and jet, while the sequences of C_u do not align [Fig. 2(a)], the sequences of C_{u^2} do align [Fig. 2(b)]. There is a possibility that C_{u^2} is at least approximately independent of the flow configuration.

DISCUSSION

For the mean rate of energy dissipation in the form of $\langle\varepsilon\rangle = C\langle u^2\rangle^{3/2}/L$, it has been traditional to define L as the velocity correlation length L_u. However, $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ is known to depend on the large-scale configuration of the flow [3, 4, 5]. We have defined L as the correlation length L_{u^2} of the local energy u^2, studied $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ for grid turbulence, boundary layer, and jet, and found a possibility that C_{u^2} does not depend on the flow configuration at least as an approximation. Not L_u but rather L_{u^2} could be proportional to the typical size L of the energy-containing eddies, so that $\langle u^2\rangle^{3/2}/L_{u^2}$ is proportional to the mean rate at which the energy is removed from those eddies to be eventually dissipated into heat.

Since our data set is limited, we have only proposed a possibility. This has to be examined in future with more extensive sets of experimental or numerical data. On the other hand, there is no theory that prefers L_u as the typical size L of the energy-containing eddies. The length L_u could rather suffer from too large scales. Especially for isotropic turbulence, Batchelor [2] pointed out that $L_u \propto \int_0^\infty k^{-1}E(k)dk$ does not represent the part of the three-dimensional energy spectrum $E(k)$ that makes the major contribution to the energy $\langle u^2\rangle \propto \int_0^\infty E(k)dk$. Thus, although we are not so confident that L_{u^2} does serve as L, it is important to persistently examine whether L_u is the most suitable definition of L.

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