# MEAN SHEAR EFFECTS AT THE CLOUD-TOP BOUNDARY

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*Summary* The interaction between a steady, horizontal mean shear and the buoyancy reversal caused by evaporative cooling at the cloud top is investigated. We use direct numerical simulations to faithfully represent the difference between the shear-free case, dominated by molecular processes, and the inviscid, shear-dominated case. Results show that turbulence intensities for typical atmospheric conditions are stronger than in the shear-free case but still too weak to break the cloud top, and that the inversion remains relatively thin.

## DEFINITION OF THE PROBLEM

The local dynamics of the cloud top is key in understanding the transition from the stratocumulus to the shallow cumulus regimes in the maritime atmospheric boundary layer over the subtropical oceans. Parametrizations thereof are needed in larger-scale models, and existing parametrizations should be improved [1]. The problem involves relatively small-scale processes, of the order of meters or tens of meters. Over this distance, temperature and water content vary rapidly between the in-cloud and upper, free-atmosphere values, defining the cloud top and the inversion that caps the turbulent boundary layer. Turbulent mixing across a density interface constitutes in its own an active area of research [2]. In the case of the cloud top, moist convection, radiation and microphysical processes further complicate the problem.
The interaction between the possible buoyancy reversal caused by the evaporative cooling and the local turbulent mixing, specifically the possibility of the so-called cloud-top entrainment instability, is one aspect of the system. Recent work using direct numerical simulations of the clod-top mixing layer have shown that buoyancy reversal alone cannot be responsible for the cloud-top break-up and regime transition because the turbulence created by this process is too weak [4]. It indicates that turbulent mixing driven by other mechanisms should be investigated. In this paper, we study the interaction between a steady, horizontal mean shear and buoyancy reversal in the cloud-top mixing layer. This idealized two layer system is sketched in figure 1. We use direct numerical simulations to faithfully represent the difference between the shear-free case, dominated by molecular processes, and the inviscid, shear-dominated case.

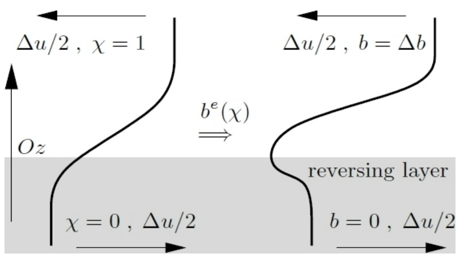


**Figure 1.** The cloud-top mixing layer consists of a region of dry, warm air, representing the free-atmosphere, on top of a region of moist, relatively cold air, representing the cloud (gray area). The mean vertical structure of the system in terms of the mixture fraction $\chi$ and the buoyancy $b$ is shown here. Each of the two layers is assumed to have a well-defined homogeneous state far enough from the cloud boundary. The buoyancy difference $\Delta b$ across the inversion measures the strength of the stratification. The velocity difference $\Delta u$ across the inversion is created, for instance, by large-scale circulations within the boundary layer.

We use a two-fluid formulation based on a continuum description of the liquid phase under equilibrium conditions. Despite this strong simplification of the cloud, a multiphase system, this formulation provides a leading-order solution to the problem of the cloud-top entrainment instability. Corrections due to the settling velocity, which moves the droplets away from the dry air above causing their evaporation, and corrections due to finite evaporation rates, which slow down the formation of cold, heavy parcels of fluid mixture, tend to weaken the strength of the buoyancy reversal; hence, the conclusion that evaporative cooling effects are too weak to break the cloud remains valid. The two-fluid formulation leads to the Boussinesq equations with a buoyancy term $b$ that is given by an analytical, non-linear function $b^e(\chi)$ (see figure 1). The mixture fraction $\chi$ is equal to the normalized enthalpy and total water content, obeys the standard advection-diffusion equation, and can be defined, without loss of generality, to be 0 inside of the cloud and 1 on the upper, cloud-free region [5]. The set of parameters defining the problem are $\{\Delta u\,, \Delta b\,, \kappa\,, \nu\,, b_s, \chi_s\}$. The Prandtl number $\nu/\kappa$ is set to one. The last two parameters characterize the buoyancy reversal in terms of the saturation anomaly $b_s < 0$ at saturation conditions $\chi_s$. We focus on the thermodynamic state measured in nocturnal marine stratocumulus during the first research flight of the DYCOMS-II field campaign, for which $b_s/\Delta b = -0.031$ and $\chi_s = 0.09$ [3].

## RESULTS

Given a dry, stably stratified shear layer with an initial thickness small enough for the Kelvin-Helmholtz instability to develop, a state is finally reached in which turbulence dissipates and molecular diffusion dominates thereafter (they are self-limiting). This final state is characterized by a bulk Richardson number $Ri_b = \delta_\omega \Delta b/(\Delta u)^2$ about 1/3, having defined the vorticity thickness as usual by $\delta_\omega = \Delta u/(\partial\langle u\rangle/\partial z)_{\max}$ (see for instance Ref. [6] and references therein). In

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the moist system that we investigate here, the buoyancy reversal will modify this scenario. However, buoyancy reversal effects are known the be small compared with the stratification ($|b_s|/\Delta b \ll 1$) and, hence, it is appropriate to conjecture a characteristic thickness $h_S = 1/3(\Delta u)^2/\Delta b$ for the inversion. Results show that indeed the sheared cloud-top develops an inversion with a depth $\delta_\omega \simeq 0.6 - 0.8 h_S$ after a relatively short transient of about $10\Delta u/\Delta b$. The difference between the dry and the moist cases is that the latter, because of the buoyancy reversal, develops a turbulent convective boundary layer inside the cloud that continuously perturbs the inversion on top. Such a state is shown in figure 2, as obtained from the direct numerical simulations. We are interested in the evolution of the system during this stage.

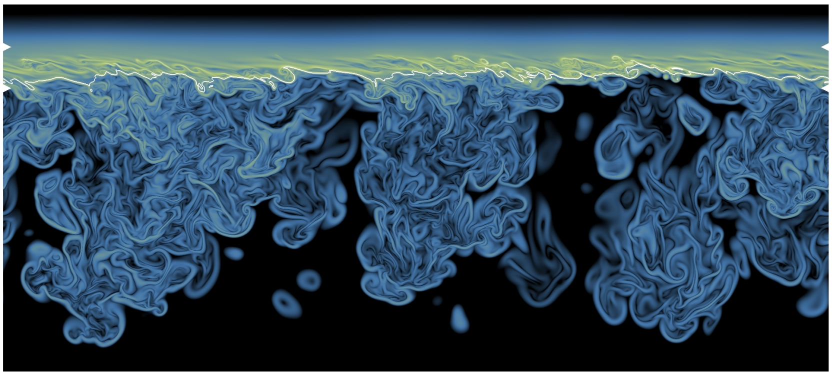

**Figure 2.** Logarithm of $|\nabla\chi|^2$ inside a vertical plane. Color code varying in the sequence black-blue-white between the values $10^{-4}/h_S^2$ and $10^4/h_S^2$. The side triangular marks correspond to $z = \pm h_S/2$ with respect to the point of maximum shear production, with $h_S = 1/3(\Delta u)^2/\Delta b$, providing therefore a measure of the inversion thickness. The thin line indicates the saturation surface $\chi(\mathbf{x}, t) = \chi_s$. The grid is $2048\times2048\times 1664$.

Qualitatively, the vertical structure of the system shown in figure 2 is very similar to that of the shear-free case discussed in Ref. [4], in particular the cloud-top remains flat and does not break. Quantitatively, there are differences. The corresponding non-dimensional parameter controlling those differences is $Re_0 = (\Delta u)^3/(\nu\Delta b)$, a reference Reynolds number. Once $Re_0$ is large enough, the mean shear, instead of the molecular transport, dominates the system. This is observed for instance in a strong peak of the shear production term $-\langle u'w'\rangle\partial\langle u\rangle/\partial z$ forming next to the inversion, significantly larger than the maximum of the buoyancy production term $\langle b'w'\rangle$ (not shown).

However, this transition of regimes from molecularly to mechanically dominated with increasing $Re_0$ occurs somehow slowly. The values that we can reach, based on our computational resources, are $Re_0 \simeq 3.3\times10^4$, which correspond to a velocity difference of about 0.5 m s$^{-1}$ for the DYCOMS-II case, where $\Delta b \simeq 0.25$ m s$^{-2}$. These values of $\Delta u$, though moderate, do cover part of the range found in the cloud top, where velocities up to 1 m s$^{-1}$ are typical. We find that there is still a contribution from molecular transport to the entrainment velocity $w_e$ of about 10%. The reason for this behavior is that the inversion remains relatively thin, $h_S$ is about 0.33 m, and this thickness, instead of the turbulent integral scale within the cloud, controls the local mixing inside the inversion. This relatively limited effect of $\Delta u$ is also manifest in the magnitude of the entrainment velocities, about 0.3 mm s$^{-1}$ for the cases $\Delta u = 0.5$ m s$^{-1}$ considered here. This is twice the entrainment velocity obtained in the shear-free case, and scaling laws suggest that $w_e$ increases proportionally to $\Delta u$, but it is still one order of magnitude smaller than measurements at the stratocumulus top.

## CONCLUSIONS

The results summarized above show that, for typical atmospheric conditions, turbulence generated locally by evaporative cooling in the presence of a steady, horizontal mean shear is stronger than in the shear-free case but still small compared to measurements, in particular too weak to break the cloud top. The inversion remains relatively thin, which causes the molecular transport to remain relevant for a non-negligible interval of $\Delta u$. For typical thermodynamic conditions at the cloud top, a cross-over value around 0.5 m s$^{-1}$ is found, for which 10% of the entrainment velocity is due to molecular effects; lower values of $\Delta u$ lead to larger contributions and *vice versa*. These results suggest that turbulent mixing generated by processes different from a horizontal mean shear are more likely to control the evolution of the cloud top.

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