PASSIVE SCALAR ADVECTION IN THE VICINITY OF TWO POINT VORTICES IN A DEFORMATION FLOW

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Summary The dynamics of passive fluid particles in the vicinity of two point vortices with arbitrary intensities, embedded in a external deformation flow, is studied. The motion of passive fluid particles is described by an 1.5 degrees of freedom dynamical system. Chaotic advection of passive fluid particles in the vicinity of these two vortices is shown to occur. The existence of impenetrable transport barriers for chaotic advection is demonstrated.

We consider the interaction of point-vortices in a two-dimensional incompressible flow, a problem which has fewer parameters as compared to a pair of distributed piecewise vortices, and the chaotic advection which can occur in the vortex vicinity. Let us consider two point vortices with arbitrary intensities in a barotropic fluid.

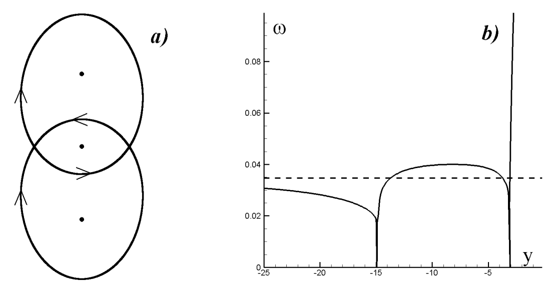

Figure 1. a) a phase portrait of the vortex system. The points in the recirculation zones correspond to stagnation points; b) the rotating frequency of the vortex pair about the elliptic critical points of the phase portrait shown in fig. 1a, versus the y coordinate. The dashed line marks the frequency of rotation of the vorticity center about the deformation center.

Figure 1a illustrates a phase portrait of the dynamical system. The phase portrait comprises phase trajectories of the first vortex in the rotating reference frame. The rotating frequency of the vortex about the elliptic critical points is shown in fig. 1b. At the coordinate origin, a singular critical point is situated. The frequency in its vicinity tends to infinity. Moving away from the singular critical point, the frequency falls dramatically to zero; this corresponds to the beginning of a recirculation zone. In the recirculation zone, the vortex rotates in the reverse direction. At the end of the recirculation zone, the frequency again turns to zero.

Let us perturb the passive scalar advection system by displacing the vortices from their stationary positions by a distance δ on the vertical symmetry axis. The vortices start to rotate along closed trajectories about the stationary positions with frequency ω_δ depending on δ (this frequency is depicted in fig. 1a for a set of the parameters; for other values of the parameters, the frequency plot has a similar shape). When the deviation δ is fixed, and when different values of S_0 are chosen, the chaotic advection domain is larger for the case of dominant rotation ($S_0 \ll \Omega_0$); conversely it is smaller when $S_0 \sim \Omega_0$. This property is due to the separatrix layers being much closer in the case $S_0 \sim 0$; therefore, they can merge at smaller perturbation values. Figure 2a shows the phase portrait of the case of $S_0 \sim \Omega_0$. The separatrix layers are situated relatively apart of each other; therefore, a stochastic sea is not established. Figure 2b illustrates the onset of a moderate stochastic sea due to the merger of two of three stochastic layers. Figure 2c shows that a stochastic sea, comprising all the stochastic layers, is established for the case of rotation predominance ($S_0 \ll \Omega_0$).

To quantify the efficiency of phase space chaotization for different values of the deviation δ, we make use of the following Lagrangian characteristic. We calculate numerically accumulated Lyapunov exponents of particle trajectories until the exponents, which correspond to regular trajectories, become stationary. A time interval, sufficient for a regular trajectory to become stationary, is chosen as 2×10^4. The accumulated Lyapunov exponents during this period are recorded as stationary Lyapunov exponents. Then, we continue calculating the accumulated exponents for a time interval of 5×10^3. If the exponent remains nearly constant during this additional time interval, then the corresponding trajectory is defined as regular. Conversely, if the exponent still evolves during this addition period, the corresponding trajectory is chaotic. This segregation of regular and chaotic trajectories is shown in fig. 3a.

Then, we uniformly distribute some 2×10^4 markers within the external separatrix of the vortex atmosphere. Making use of the procedure presented above, we estimate the proportion of chaotic trajectories of the initially distributed markers. Figure 3b shows plots of the proportion of chaotic trajectories versus the deviation value δ. It is seen that chaotization is more efficient when rotation dominates over deformation.

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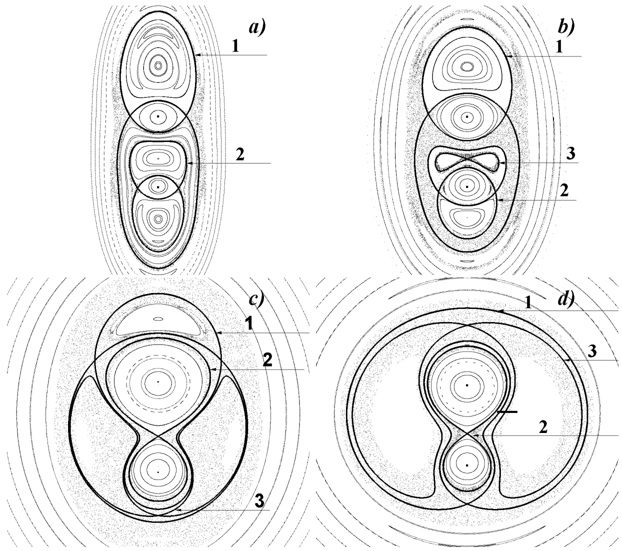

Figure 2. Poincaré sections of the system as $\delta = 0.3$. The bold lines correspond to the separatrices of the stationary motion.

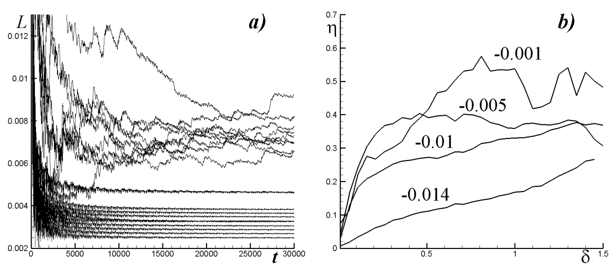

Figure 3. a) accumulated Lyapunov exponents of regular and chaotic trajectories versus the time of the accumulation. The regular trajectories become stationary quite fast, chaotic trajectories, on the contrary, have no stationary values; b) the proportion of chaotic trajectories for the characteristic values of the shear specified in the figure, with respect to the deviation δ of the vortex position.

CONCLUSION

Though the deformation flow we considered is stationary, the motion of passive fluid particles is described by a nonintegrable 1.5 degrees of freedom dynamical system. In this case, the additional "half" degree of freedom appears because the motion of the vortices about their stationary positions is periodic. The periodic motion of the vortices plays the role of a periodic perturbation for the system describing the passive particle dynamics. Therefore chaotic advection of passive fluid particles in the vicinity of the vortex pair can occur.

When the vortices are displaced from their stationary position, the existence of impenetrable transport barriers for chaotic advection has been shown. These barriers are destroyed when stochastic layers merge; such layers widen as the deviation increases. Also, these barriers disappear when separatrix reconnections occur, as stochastic layers are very close.