LABORATORY EXPERIMENTS ON THERMAL INSTABILITIES IN A YIELD-STRESS FLUID

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Summary We present laboratory experiments on the development of thermal plumes out of a localized heat source in Carbopol, a yield-stress and shear-thinning fluid. The experiments show that the development of an instability is strongly depending on the Yield parameter Y_0, the inverse Bingham-number. We investigate the onset-criterion, derive a scaling law for the onset-time and describe the spatio-temporal evolution of such an instability.

INTRODUCTION

Thermal convection in non-Newtonian fluids occurs in many different fields, from engineering (glass-production, food industry) to geoscience (planetary mantles, lava lakes, icy satellites), but is still not very well understood. From a theoretical point of view, the difficulty is that the viscosity of these fluids approaches infinity when the motion amplitude vanishes. Therefore convective instabilities cannot grow from a static conductive state submitted to infinitesimal perturbations as in the classical Rayleigh-Bénard configuration. e.g. [1]. For this reason, we study the development of thermal plumes out of a localized heat source (Peltier element) in Carbopol. Studying the rheology shows that this fluid presents a yield stress and is shear thinning.

EXPERIMENTAL SETUP

The experiment is placed in a plexiglas-tank with a ground-area of 20 cm $\times$ 20 cm, . The heat source is a Peltier-element, which is provided with constant electric power P and is placed in the the center of the bottom of the box with a small copper-plate on top of it. The temperature of the copper-plate is measured every five seconds with a thermocouple. We illuminate our fluid with a laser-sheet of a wave-length at 532 nm. Using thermochromic-liquid-crystals (TLCs are reflecting the light of a certain wavelength at a certain temperature) enables us to see isotherms. Combining this technique with particle-image-velocimetry (PIV) allows us to measure simultaneously temperature and velocity fields without disturbing the flow, [2].

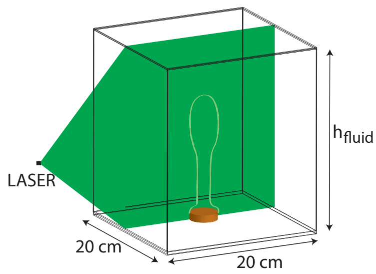

Figure 1. Experimental Setup

The fluid used is a mixture of Carbopol (ETD2050), distilled water and Glycerol. The rheology of such a fluid can be described by a Herschel-Bulkley model, e.g. [3], [4]:

$$\sigma = \sigma_0 + k_\nu \, \dot\gamma^n \tag{1}$$

where σ is the stress, σ_0 the yield stress, $\dot\gamma$ the shear rate, n the shear thinning parameter and k_ν the consistency.

RESULTS

The fluid around the heater forms a hot pocket which is therefore less dense and tends to rise. Due to the properties of the fluid, the formation of a plume is only possible if the thermally induced stress σ_{th} overcomes the yield stress [5]. At constant power, the hot pocket evokes a thermal stress which can be formulated as:

$$\sigma_{th} \propto \frac{\alpha \, \rho \, g \, P_{th}}{k} \tag{2}$$

where α, ρ and k are properties of the fluid, namely thermal expansivity, density and thermal conductivity respectively, whereas P_{th} is the thermal power delivered by the Peltier-element and g the gravity. This yields the definition of the yield parameter Y_0 as the ratio of the thermally induced stresses and the yield stress:

$$Y_0 = \frac{\alpha \, \rho \, g \, P_{th}}{k \, \sigma_0} \tag{3}$$

Depending on the yield-parameter 3 different regimes develop, cf. **figure 2 A**: i) $Y_0 < Y_{0c1} = 150 \pm 10$ stable conductive temperature profile without any convection, ii) $Y_{0c1} < Y_0 < Y_{0c2} = 500 \pm 10$ small convection-cell confined around the heater, iii) $Y_0 > Y_{0c2} = 500 \pm 10$ thermal instability rises through the fluid. The instability rises episodically before it reaches a stationary regime.
Assuming a constant power-supply, heat conservation implies

$$P_{th} \, t = \rho \, C_p \, \pi \, r^2 \, \delta(t) \, \Delta T(t) \tag{4}$$

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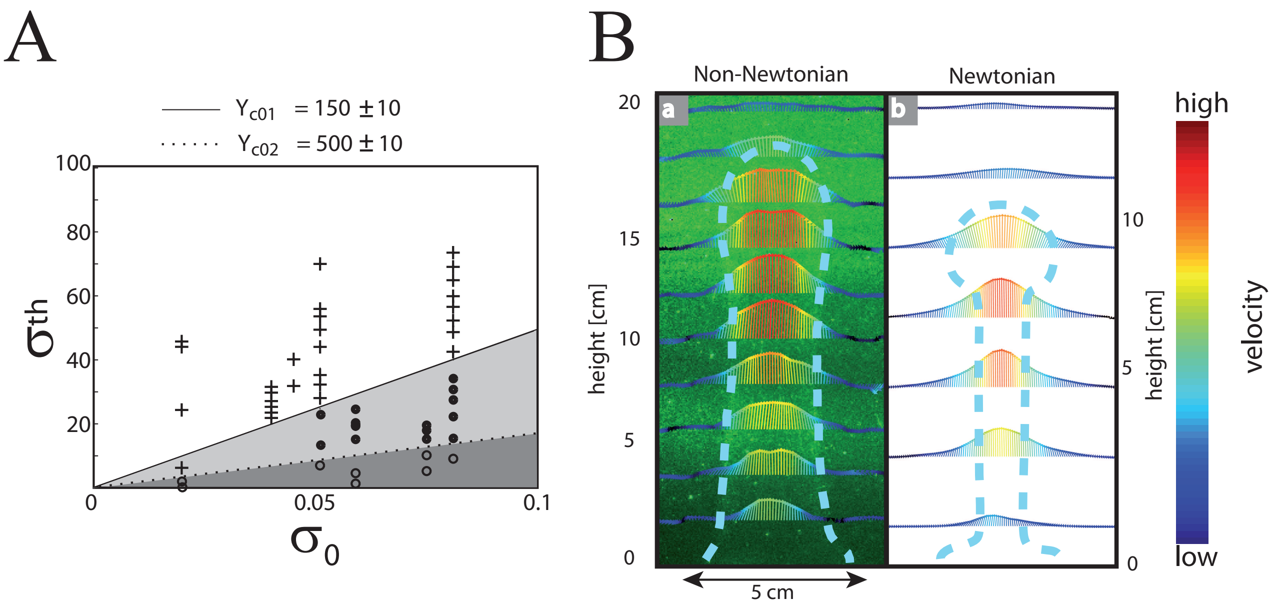

Figure 2. **A**: experimental data, σ_{th} vs. σ_0. *Circles*: no convection develops. *Dots*: small convecting cell confined around the heater. *Plus*: thermal instability rises through the fluid. **B**: Blue line highlights 24° C-isotherm, arrows: velocity-field for *a*: thermal instability in Carbopol and *b*: thermal instability in a Newtonian fluid (sugar syrup, image courtesy of F. Touitou).

where C_p represents the specific heat capacity at constant pressure, r the radius of the copper-plate, δ the thickness of the thermal boundary layer and ΔT the temperature-difference between the hot pocket and the ambient fluid. Furthermore in the conductive stage prior to the onset the thermal boundary layer grows as $\delta(t) = c\sqrt{\kappa t}$ where c is a constant and κ is the thermal diffusivity. Therefore i) assuming that at onset $\sigma_{th} = (500 \pm 10)\,\sigma_0$ and ii) equating (3) and (4) yields a critical onset time, which is given as:

$$\tau_c \propto Y_0^{-2} \tag{5}$$

and can be reproduced by the experiments.

Figure 2 B *a* shows a typical thermal instability in a non-Newtonian fluid. The shape of the thermal anomaly (highlighted by the dotted blue line representing the 24° C-isotherm) is more like a finger and therefore strongly differs from the classical mushroom-shape observed in Newtonian fluids, cf. **figure 2 B *b***. The velocity-field (*a*) shows that the deformation is limited to the thermal anomaly. Comparing the velocity profile to the one of a thermal instability in a Newtonian fluid (for comparison *b*) we can see that the velocity profile is much more flat for the non-Newtonian fluid. Therefore we can conclude that the velocity-field inside the thermal instability in a yield stress fluid is a transition from a gaussian velocity-field in a Newtonian fluid to a more plug-flow-like velocity field, as observed for yield stress fluids in tubes, cf. e.g. [6].

CONCLUSIONS

Our experiments show that the development of a thermal instabilities in a yield stress fluid is different from those in Newtonian fluids. The governing parameter of the system is the yield-parameter which controls if and when a thermal instability develops. The morphology of the temperature anomaly looks more like a finger than the classical mushroom-shape observed in Newtonian fluids. The shape of the velocity field looks like a transition from the Gaussian shape observed in Newtonian fluids to the flat plug-flow often observed for yield-stress fluids.

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