FINITE REYNOLDS NUMBER EFFECTS FOR DENSE SUSPENSIONS IN SHEAR FLOW

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Summary Rheological properties of dense suspensions are of definitive interest for application and from a fundamental point of view. Crucial aspects relate to the macroscopic effective viscosity of the suspension. It increases with the volume fraction up to diverge at the maximum volume fraction (jamming). Shear-thinning or shear-thickening behaviors are observed when increasing the flow shear rate. The former, occurring at low shear rate, amounts to viscosity decreasing with the shear and it is associated with a finite Peclet number (noticeable effect of thermal fluctuations). At higher shear rate, the opposite behavior, shear-thickening, takes place and it is associated with finite particle Reynolds number. The aim of this work is to investigate shear-thickening in a non-Brownian suspension of spherical rigid particles, highlighting the particle spatial organization of the particles and its rheological influence. The dataset is obtained from simulations performed by an Immersed Boundaries Method for particle tracking coupled with a Navier-Stokes solver.

INTRODUCTION

Suspensions of solid particles in a viscous liquid are frequently found in several contexts, e.g. biological systems or pharmaceutical and industrial processing. Here we focus on dense suspensions where the average separation distance between particles is smaller than the particle size, see [1]. This limit is achieved when the volume fraction ϕ is larger than 10%. Whereas the dilute limit is well understood (e.g. [2]), the rheology in the dense limit still leaves open questions. The solid phase microstructure is crucial to determine the rheological features of these complex flows. In particular different microstructures may lead to shear thinning or shear thickening, see [3]. The former amounts to a decrease of the effective viscosity of the suspension at fixed volume fraction when increasing the shear rate; the latter, on the contrary, it is an increase of the effective viscosity with the shear rate. Both these phenomenologies are found in dense suspensions. Shear thinning usually occurs when the shear rate is below a critical value while shear thickening above that critical value. This macroscopic change in the suspension rheology is associated to a reorganization of the particle microstructure. In the shear-thinning regime, the microstructure is a balance between hydrodynamic forces induced by the flow and the particle Brownian motions. For shear-thickening to occur the particles are non-Brownian, and, typically, the particle Reynolds number is finite, leading to the formation of the so-called particle hydroclusters. Because a characterization of the particle hydroclusters when inertial effects are important is still lacking, the aim of the present work is to analyze a suspension of spherical rigid and neutrally-buoyant particles to better understand the shear-thickening. To this purpose, direct numerical simulations of dense suspensions in Couette flow with different Reynolds numbers and volume fractions are performed. The numerical tool is based on finite differences for the Navier-Stokes solver combined with an Immersed Boundary Method to model the interaction between fluid and particles [4]. As expected the effective viscosity is found to increase both increasing the Reynolds number and the volume fraction. The peculiar features of the particle microstructures are investigated to address their effects on the macroscopic rheology.

METHODOLOGY

The simulations have been performed by a numerical code that fully describes the coupling between the solid and fluid phases. The incompressible Navier-Stokes equations are discretized by second order finite differences on a staggered mesh in space and a third order accurate Runge-Kutta scheme in time. The particles are evolved by a Lagrangian algorithm that

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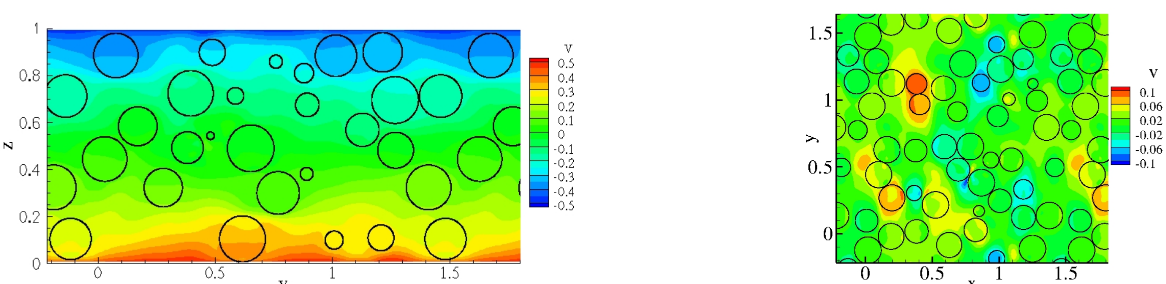

Figure 1. Contours of streamwise velocity v with particles superimposed, $Re_p = 1$ and $\phi = 0.3$; left panel: streamwise-wallnormal $x - z$ plane, right panel: streamwise-spanwise $x - y$ middle plane.

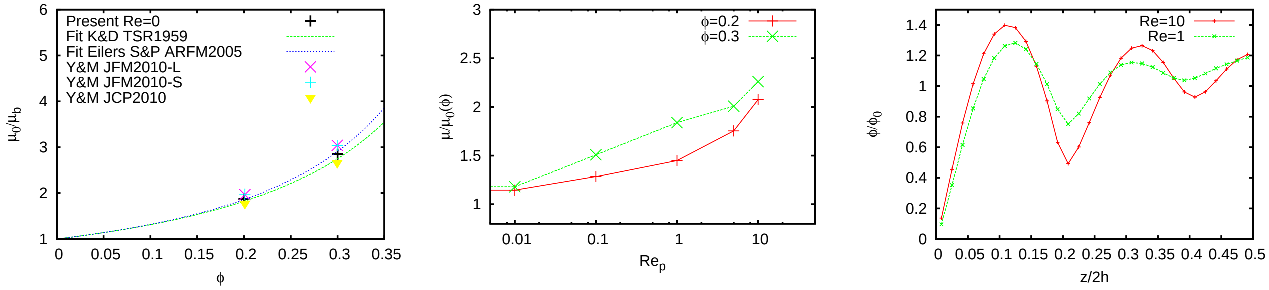

Figure 2. From left to right: Effective viscosity μ_0 normalized by the bulk value μ_b vs ϕ at $Re_p = 0$ compared with models [3] Eilers $(1+1.25\phi/(1-\phi/0.65))$, Krieger & Dougherty $(1-\phi/0.65)^{(-2.5\cdot 0.65)}$ and numerical data [5]; Normalized effective viscosity $\mu/\mu_0(\phi)$ vs Re_p; Mean local volume fraction $\phi(z)$ vs z at $\phi = 0.3$ for $Re_p = 1$ and $Re_p = 10$.

solves the linear and angular momentum equations. The coupling between the two phases is directly achieved by using an Immersed Boundary Method. Lubrication models are also used to correctly reproduce the interaction between particles when their gap distance is smaller than the mesh size. Periodic boundary conditions are imposed in the streamwise x and spanwise y directions. Couette flow is obtained prescribing constant opposite velocity at the wall $z = 0$ and $z = 2h$. The code was fully validated against several classic test cases, see [4] for more details on the numerical implementation.

Simulations were performed in a domain of length $16\,a$ in the streamwise and spanwise direction and $2h = 10\,a$ in the wall-normal one, with a the particle radius. A cubic mesh is adopted for all the simulations with eight points per particle radius. Non-Brownian spherical particles with same density as the fluid are considered. Two volume fractions, $\phi = V_s/V_f = 0.2$ and $\phi = 0.3$, and several particle Reynolds numbers, $Re_p = \rho S\, a^2/\mu_b = 0.01 \div 10$ have been considered, with S the bulk shear rate across the walls, μ_b and ρ the fluid viscosity and density.

Snapshots of a typical simulation are given in figure 1 where instantaneous streamwise velocity cuts are presented with the particle boundaries superimposed. Particles seem to align preferentially in the compression direction (45^0 in the $x-z$ plane), see left panel. Particle clusters are evident in the homogeneous $x-z$ plane, see right panel of figure 1.

RESULTS & FINAL REMARKS

Data from simulations of dense suspensions at zero Reynolds number were compared against experimental and numerical data at different volume fractions. As expected, the effective viscosity $\mu_0(\phi)$ increases with the volume fraction, see left panel of figure 2, in excellent agreement with semi-empirical models [3] and numerical data [5] in literature.

The effects of Reynolds number on the effective viscosity is addressed in the middle panel of figure 2. The viscosity increases monotonically with the Reynolds number. Data show that at $Re_p \approx \mathcal{O}(10^{-2})$ the viscosity is still larger than the reference zero Reynolds number value, so the effect of fluid inertia is relevant also at these very low values of the particle Reynolds number. Since the increase of the Reynolds number corresponds to an increase of the actual overall shear rate S in experiments, this behavior indicates shear thickening. This phenomenology is connected to the particle microstructure, relative position and orientation. Dealing with an inhomogeneous flow the mean particle concentration also influences the system and gives interesting informations. The right panel of figure 2 provides the mean local volume fraction $\phi(z)$ normalized with the bulk value ($\phi = 0.3$) vs the wall-normal coordinate z for two cases. Similar to liquid molecular dynamics, particle distribution shows layering close to the wall [1, 5]. The concentration peak occurs near the wall in all cases. The largest is the Reynolds number, the highest is the peak value and closest to the wall it occurs. In particular at $\phi = 0.3$ and $Re_p = 10$ it is 40% larger than the bulk value and occurs at a distance from the wall of $1.1\,a$, while at $Re_p = 1$ is only 28% larger at $1.25\,a$. Consistently, at a distance slightly larger than $2\,a$ the minimum volume fraction occurs, and the lowest value pertains to $Re_p = 10$ with almost half of the bulk volume fraction. Away from the wall, the layering is still stronger for the highest Reynolds number.

This behavior suggests that increasing the Reynolds number (or equivalently the shear rate) particles tend to arrange in layers parallel to the wall moving at different velocities with few particles shifting among them. The analysis will be completed presenting data on the particle microstructures (e.g. radial distribution function) and on cluster dimension.

References

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