A THEORETICAL STUDY OF THE EFFECT OF POLYMER CONCENTRATION ON TURBULENT DRAG REDUCTION

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Summary A recent theory has been developed for understanding turbulent drag reduction by polymers in wall-bounded flows. In this theory, which is based on the momentum and energy balance, the effect of the polymers is shown to give rise to a position-dependent effective viscosity. Based on this theory, we have worked out the effect of polymer concentration in the phenomenon. In particular, we have calculated the profiles of the Reynolds stress and their dependence on the concentration of the polymers. We have also calculated the percentage of drag reduction and how it varies with concentration. Our theoretical results show good qualitative agreement with experimental observations.

BACKGROUND

The phenomenon of polymer drag reduction was discovered in experiment by Toms in 1948 [1]. It was found that the addition of a small amount of polymer can reduce the frictional drag in turbulent wall-bounded flows by up to 80%. Consider the turbulent flow of a fluid in a channel of half height L with x, y and z being the streamwise, wall-normal and spanwise directions respectively. The reduction in frictional drag is characterized by a decrease in the friction factor $f \equiv \Delta p L/(\frac{1}{2}\rho U_{av}^2 l)$ where Δp is the pressure drop across a streamwise length l of the channel, ρ is the density of the polymer solution. $U_{av} \equiv (L)^{-1}\int_0^L V(y)dy$ is the bulk velocity where y is the distance from the wall and $V(y)$ the time-averaged streamwise velocity profile. Flow in a fluid with polymer additives is described by the following equations of motion,

$$\rho(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}) = -\nabla p + \eta_s \nabla^2 \mathbf{U} + \nabla\cdot\mathbf{\Sigma}; \qquad \nabla\cdot\mathbf{U} = 0 \tag{1}$$

where η_s is the dynamic viscosity of the solvent and $\mathbf{\Sigma}$ the additional stress tensor due to the polymer. The form of $\mathbf{\Sigma}$ depends on the polymer model. We focus on the FENE-P model [2] of flexible polymers in which the polymer chain is modeled as a dumbbell connected by a spring of equilibrium length ρ_0 and maximum extension ρ_m. By decomposing the fluid velocity $\mathbf{U}$ into the sum of its time-average $V(y)\hat{\mathbf{x}}$ and fluctuations $\mathbf{u}$ and then averaging, Eq. (1) leads to two equations corresponding to the momentum and energy balance that contain the mean shear rate $S = dV(y)/dy$, the Reynolds stress $W = -\langle u_x u_y \rangle$, and correlations involving the polymer stress. A recent theory has been developed [3] to estimate these correlations at large Weissenberg number based on order-of-magnitude arguments showing that the effect of polymers can be represented by a position-dependent effective viscosity ν_{eff}. The resulting equations are:

$$\nu_{eff}S^+ + W^+ = 1 - \frac{y^+}{L^+}; \qquad S^+ = \nu_{eff}(\frac{\Delta}{y^+})^2 + \frac{\Delta^3 b\sqrt{W^+}}{a^3 y^+}; \qquad \nu_{eff} = 1 + \frac{\phi\Delta}{a}\frac{\sqrt{W^+}}{S^+ y^+} \tag{2}$$

where $\phi \equiv (\rho_m/\rho_0)^2 \eta_p/\eta_s$ in which η_p is the polymer contribution to the zero-shear viscosity and a, b, Δ are parameters in the theory chosen to reproduce experimental results for the Newtonian flow without polymers. Here the superscript $+$ denotes the dimensionless wall units where all length scales and velocity scales are normalized using the friction velocity u_τ and $\nu_s \equiv \eta_s/\rho$ according to $y^+ \equiv y u_\tau/\nu_s$ and $V^+ \equiv V/u_\tau$ respectively.
Once ϕ and the half-width of the channel L^+ are given, the mean velocity profile $V^+(y^+)$ and the Reynolds stress profile $W^+(y^+)$ can be calculated [4] from the theory. The bulk velocity U_{av}^+ and the friction factor f can then also be computed. Similar calculations can be conducted for turbulent flow in a pipe of radius L with the modifications: the bulk velocity $U_{av} = (\pi L^2)^{-1}\int_0^L V(y)2\pi(L-y)dy$ and the friction factor $f = \Delta p L/(\rho U_{av}^2 l)$.

RESULTS

Using the theory, we have calculated the percentage drag reduction and the Reynolds stress profile. The percentage drag reduction is defined as,

$$DR \equiv (1 - \frac{f}{f_N}) \times 100\% \tag{3}$$

where f_N is the friction factor of the Newtonian flow without polymers evaluated at the same Reynolds number $Re \equiv 2LU_{av}/\nu_s$. As shown in Fig. 1a, DR generally increases with ϕ and saturates at sufficiently large ϕ. Both the saturated value and the value of ϕ needed for saturation increases with the Reynolds number. We have also found that the ϕ dependence of DR at high Reynolds number can be collapsed by rescaling DR by its maximum value at a fixed Reynolds number DR_m and ϕ by its intrinsic value $[\phi] \equiv DR_m/(\lim_{\phi\to 0} DR/\phi)$ [5]. The collapsed curve is shown in Fig. 1b. As seen in the graph, the collapsed master curve is well approximated by the functional form $f(x) = \alpha x/(1+\alpha x)$ with the fitting parameter $\alpha \approx 1.38$.

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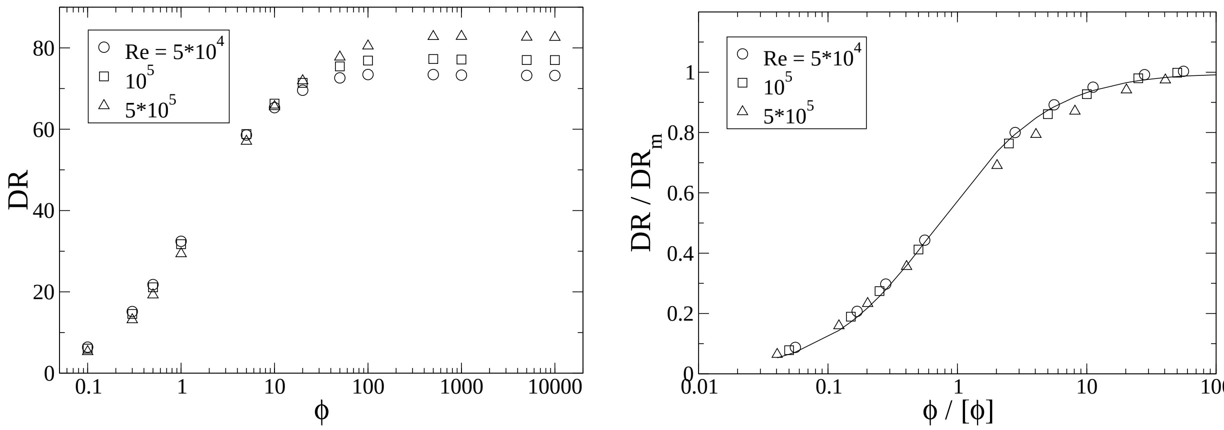
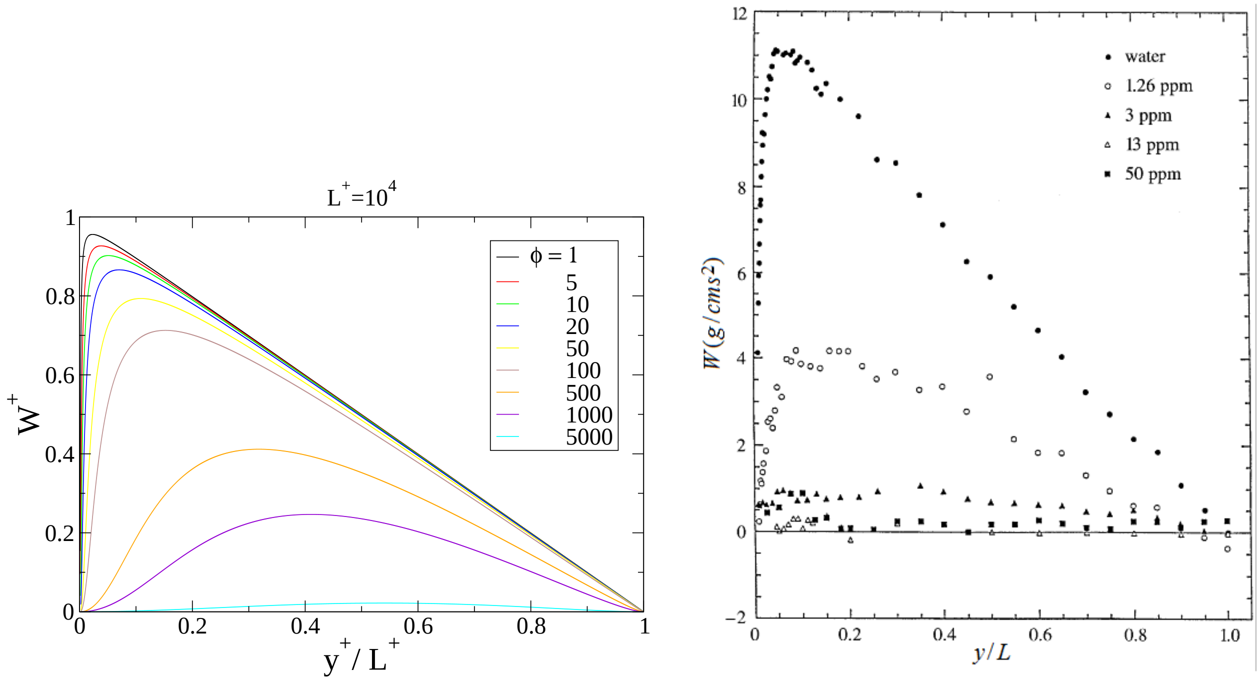
Theoretical results of the Reynolds stress profiles at different values of ϕ are shown in Fig. 2a. The theoretical profiles show qualitative agreement with the experimental profiles shown in Fig. 2b [6].



(a) (b)

Figure 1: (a) DR against ϕ from the theory at different Reynolds number in pipe flow. (b) Collapsed master curve by rescaling DR and ϕ. Solid line is the function $f(x) = \alpha x/(1 + \alpha x)$ where $\alpha \approx 1.38$.



(a) (b)

Figure 2: (a) Theoretical Reynolds stress profiles at different ϕ in channel flow. (b) Experimental Reynolds stress profiles of an aqueous solution of the copolymer of polyacrylamide and sodiumacrylamide in channel flow taken from Ref. [6].

References

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