GAS-KINETIC UNIFIED ALGORITHM FOR THREE-DIMENSIONAL COMPLEX FLOWS ON SPACECRAFT RE-ENTRY USING BOLTZMANN MODEL EQUATION

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Summary The unified Boltzmann model equation describing the gas transfer phenomena for various flow regimes is presented. The discrete velocity ordinate method and numerical quadrature technique are developed to simulate the multi-scale flows. The gas-kinetic finite difference scheme is constructed to directly solve the discrete velocity distribution functions. The gas-kinetic unified algorithm is established for the complex flows from free-molecule rarefied to continuum regimes on spacecraft re-entry.

The Boltzmann equation [1] can describe the molecular transport phenomena for the full spectrum of flow regimes and act as the main foundation for the study of complex gas dynamics. However, the difficulties encountered in solving the full Boltzmann equation are mainly associated with the nonlinear multidimensional integral nature of the collision term, and exact solutions of the Boltzmann equation are almost impractical for the analysis of practical complex flow problems up to this day [2]. From the kinetic-molecular theory of gases, numerous statistical or relaxation kinetic model equations resembling various order of moments of the original Boltzmann equation have been put forward [3~5], and the numerical methods of the kinetic models of the original Boltzmann equation have been set forth [5~7].

Based on the Boltzmann model equation describing the gas transfer phenomena for various flow regimes, the gas-kinetic unified algorithm for flows from rarefied transition to continuum has been successively presented [8~10] and applied to the one-dimensional, two-dimensional and three-dimensional gas flows with a wide range of Mach numbers. The velocity distribution function equation for describing the complex multi-scale flows on spacecraft re-entry covering various flow regimes can be presented without the interaction of external force, as follows

$$\frac{\partial f}{\partial t}+\vec{V}\cdot\frac{\partial f}{\partial \vec{r}}=\nu(f^N-f)\,, \tag{1}$$

$$f^N = f_M\cdot[1+(1-\Pr)\vec{c}\cdot\vec{q}\left(2c^2/T-5\right)/(5PT/2)]\,, \tag{2}$$

$$f_M=n/(\pi T)^{3/2}\exp[-c^2/T]$$

$$\nu=\frac{2\alpha(5-2\omega)(7-2\omega)}{5(\alpha+1)(\alpha+2)\sqrt{\pi}}\cdot\frac{1}{Kn}\cdot nT^{(1-\chi)}\,,\quad Kn=\lambda_\infty/L\,. \tag{3}$$

The equation (1) represents the time dependence of the molecular velocity distribution function on position space $\vec{r}=(x,y,z)$ and velocity space $\vec{V}=(V_x,V_y,V_z)$ and provides the statistical description of gas flows from the level of the kinetic theory of gases. All of the macroscopic flow variables of gas dynamics in consideration, such as the density of the gas ρ, the flow velocity $\vec{U}$, the temperature T, the pressure p, the viscous stress tensor τ and the heat flux vector $\vec{q}$, can be evaluated by the moments of the velocity distribution function over the velocity space [1,2,9].

In order to replace the continuous dependency of f on the velocity space, the discrete velocity ordinate method [8,9] can be employed to discretize the finite velocity region removed from $\vec{U}$. The single velocity distribution function equation can be transformed into hyperbolic conservation equations with nonlinear source terms at each of discrete velocity ordinate points. In view of the unsteady characteristics of molecular convective movement and colliding relaxation, the time-splitting numerical method can be adopted so that the discrete velocity distribution function equation can be decomposed into the colliding relaxation equations with the nonlinear source terms and the convective motion equations in three directions of the position space. The integration of the source term of colliding relaxation is solved by the second-order Runge-Kutta method, and the NND-4(a) scheme based on primitive variables is applied to the convective equations, respectively. The second-order finite difference scheme directly solving the six-dimensional discrete velocity distribution functions can be constructed.

Once the discrete velocity distribution functions $f_{\sigma,\delta,\theta}$ are solved, the macroscopic flow moments at any time in each point of the physical space can be computed and updated by the appropriate discrete velocity quadrature method, in which the evaluation points of the integration are related to the determined discrete velocity ordinate points. In this study, the new Gauss-type integration method with the weight function $2/\pi^{1/2}\exp(-V^2)$ will be further developed and applied to attack hypersonic flows with different Mach numbers.

In the gas-kinetic algorithm, it is the most time-consuming process that the discrete velocity distribution functions are numerically solved at each of discrete velocity ordinate points, which needs to use the six-dimensional array to store the information of the discrete velocity distribution functions with the six-layer loop operations. To resolve the difficulty on the vast computer memory and runtime needed by the current method in solving three-dimensional complex flows and to well exploit the power of massively parallel computers with rapid development, the multi-processing strategy and parallel implementation technique suitable for the gas-kinetic numerical algorithm has been investigated by using the technique of domain decomposition.

To validate the reliability of the present gas-kinetic unified algorithm in solving the three-dimensional problems from free-molecule flow to rarefied transition and continuum flows, the complex flows around sphere and spacecraft shape are computed and studied with different Mach numbers $3 < M_\infty < 16$ under various Knudsen numbers $0.0001 < Kn_\infty < 8.3$ and angles $0 \le \alpha \le 30°$ of attack. Table 1 shows the computed results of the drag coefficients past sphere related to $3.8 < M_\infty < 4.3$, $0.006 < Kn_\infty < 0.107$, and the corresponding free-stream Reynolds number $50 < \mathrm{Re}_\infty < 1000$ under the cases of ten with the comparison of the experimental data [12]. The computed drag coefficients are in good agreement with the experimental data, the relative differences between the calculations and the experiments are of the order of 0.31%~5.53%, as indicates that the present algorithm has good capability in computing the aerodynamics of rarefied transition and near-continuum flows even if the coarse spatial mesh system is used. Figure 2 shows the computed results of the drag coefficients for eight cases of $8 < M_\infty < 13$, $0.005 < Kn_\infty < 8.266$, $1.5 < \mathrm{Re}_\infty < 3950$, and $58Km < H < 85Km$ past sphere. The comparison between the computed results of the drag coefficients and the experimental data [13] is in good agreement.

Tab.1 Comparison of drag coefficients of sphere from computation and experiment ($3.8 < M_\infty < 4.3$, $0.006 < Kn_\infty < 0.107$)

$d_S(m)$	0.0191	0.0381	0.0476	0.0381	0.0476	0.0476	0.0572	0.1143	0.1524	0.1523
Kn_∞	0.1071	0.0550	0.0447	0.0350	0.0319	0.0203	0.0163	0.0094	0.0079	0.0064
$H(Km)$	76.525	76.732	76.841	73.522	74.466	71.113	70.828	71.913	72.765	71.180
M_∞	3.865	3.865	3.863	4.169	4.096	4.322	4.324	4.275	4.229	4.322
$C_{d,Exp}$	1.713	1.502	1.452	1.337	1.336	1.275	1.229	1.227	1.206	1.177
$C_{d,Cal}$	1.743	1.491	1.457	1.411	1.389	1.279	1.255	1.233	1.212	1.211
$Error(\%)$	1.75%	0.73%	0.34%	5.53%	3.97%	0.31%	2.12%	0.49%	0.50%	2.89%

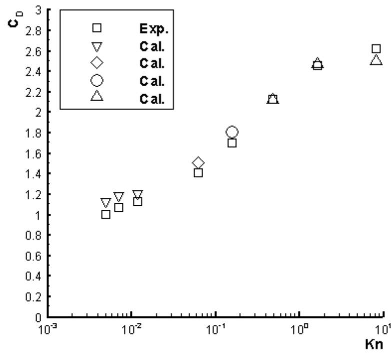
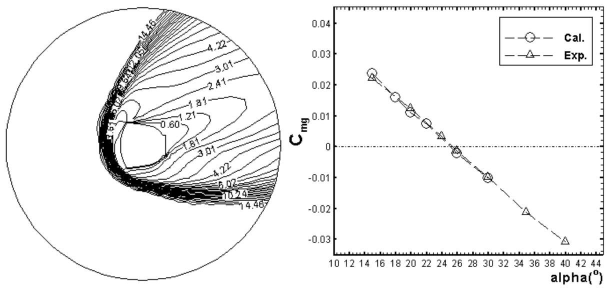


Fig.2 Drag coefficients for hypersonic flow past a sphere.

The three-dimensional complex flows past the spacecraft shape are computed with various Knudsen numbers $0.006 < Kn_\infty < 5$ and Mach numbers $4 < M_\infty < 16$. Figure 3(a) illustrates the Mach number contours past the body with the flying angle of attack $\alpha = 26°$, $Kn_\infty = 0.0063$, $M_\infty = 15.587$, $H = 88.34km$, $\mathrm{Re}_\infty = 3729.15$ and $T_w/T_0 = 0.5435$. Figure 3(b) presents the pitching moment coefficient C_{mg} relative to the centre of mass as a function of angle of attack α. It can be shown from the comparison that the present computations are in good agreement with the experiments [14], where the computed trim angle of attack is $\alpha_{Cal} = 25.06°$ and the experimental measurement is about $\alpha_{Exp} = 25.39°$. The above computations are nicely consistent with the theoretical prediction and the experiment, and affirm the flow phenomena during spacecraft re-entering Earth's atmosphere.



(a) Mach number contours $\alpha = 26°$ (b) Pitching moment coefficient

Fig.3 Hypersonic flow past spacecraft for $Kn_\infty = 0.0063$, $M_\infty = 15.587$.

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